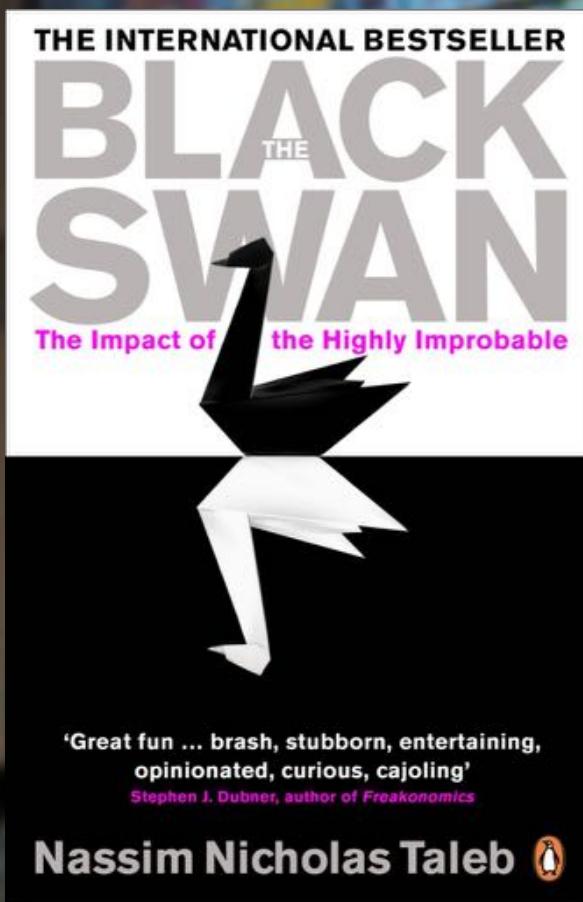




value  
Extreme  $\wedge$  Statistics, Black Swans, high-e low-p risk  
Latham Stack, MS, CIA, CGAP



# Nassim Taleb's The Black Swan



# Risk-assessment

Cause

## Recommendation

Effect

2013  
EDITION

- Definition of Internal Audit
- Code of Ethics
- International Standards



Criteria

GAO

August 2011

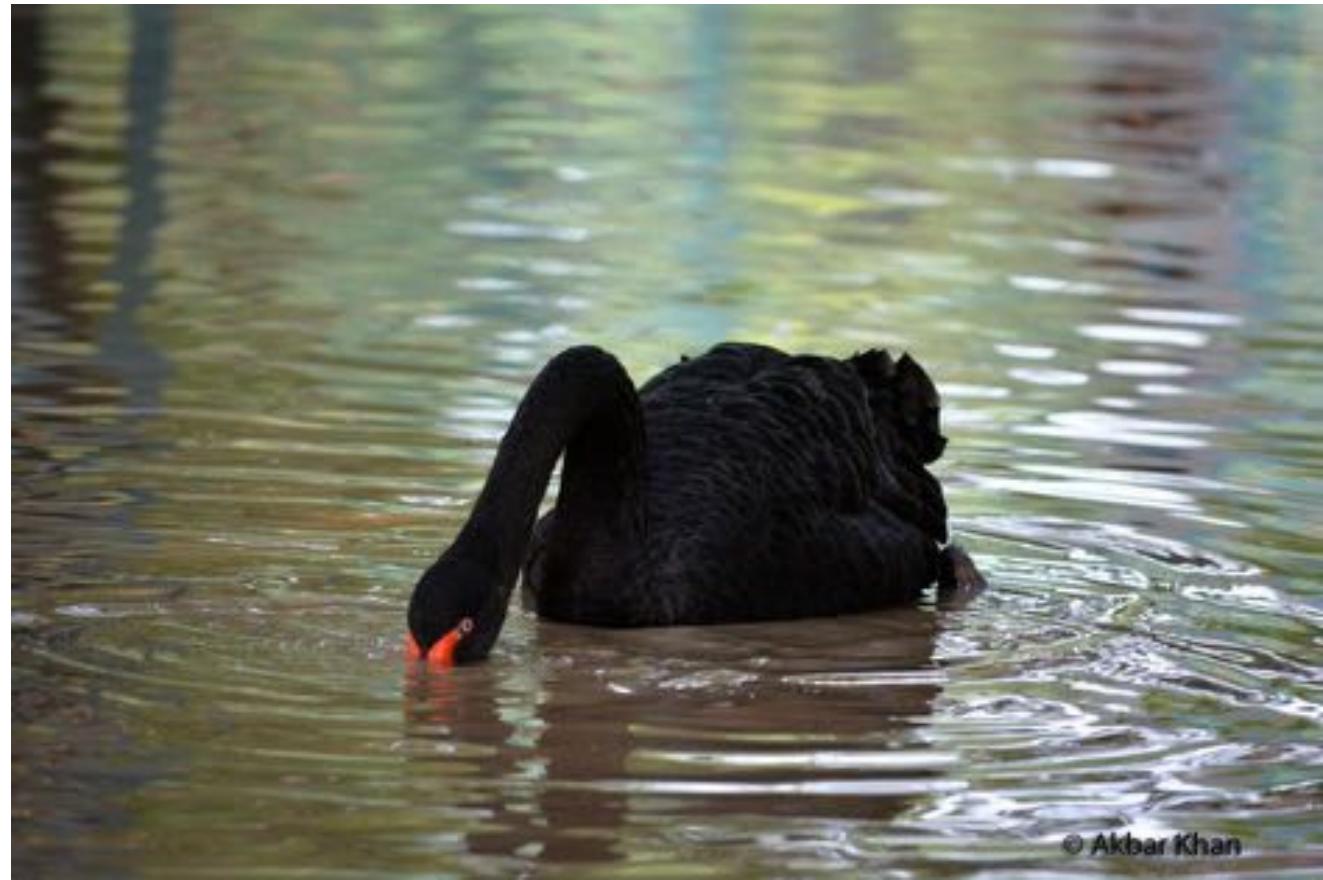
United States Government Accountability Office  
By the Comptroller General of the  
United States

2011 Internet Version

# Outline:



## Nassim Taleb's The Black Swan



# Outline:



**Risk, complex systems, failure**

**Nassim Taleb's The Black Swan**



# Outline:



## Extreme Value Analysis

Nassim Taleb's The Black Swan  
Risk, complex systems, failure

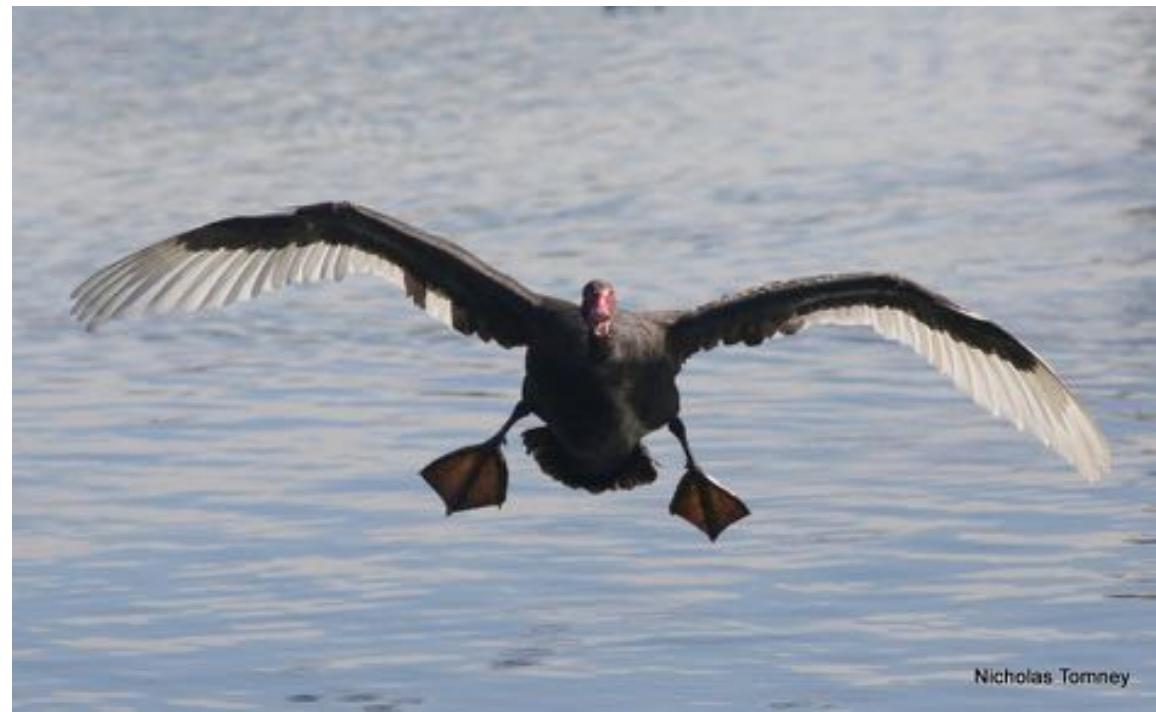


# Outline:

## EVA in program auditing I: quantifying exposure & $p$

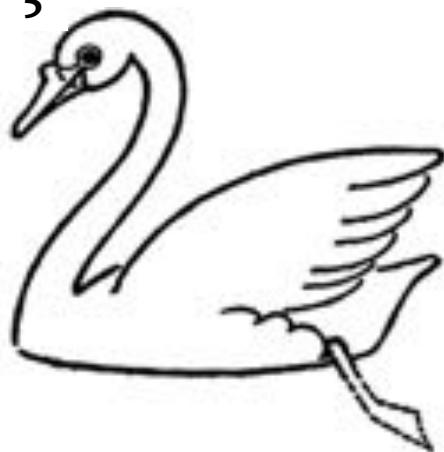


Nassim Taleb's The Black Swan  
Risk, complex systems, failure  
Extreme Value Analysis



# Outline:

5



## EVA in program auditing: II risk mitigation

Nassim Taleb's The Black Swan  
Risk, complex systems, failure  
Extreme Value Analysis  
EVA in practice, program auditing I



# Outline:

## Wrap and Q & A

6



**Nassim Taleb's The Black Swan**

**Risk, complex systems, failure**

**Extreme Value Analysis**

**EVA in practice, program auditing I**

**EVA in practice, program auditing II**

**Wrap, Q&A**





# Risk, complex systems, failure, lessons from accident investigation and safety engineering



# FLIRTING WITH DISASTER

WHY ACCIDENTS ARE  
RARELY ACCIDENTAL

**SYSTEMANTICS**  
How Systems Work and  
Especially How They Fail

TITANIC

# Normal Accidents

Living with High-Risk Technologies

Charles  
Perrow

Scott D. Sagan  
**The Limits of Safety**

Organizations, Accidents,  
and Nuclear Weapons

# Drift into Failure

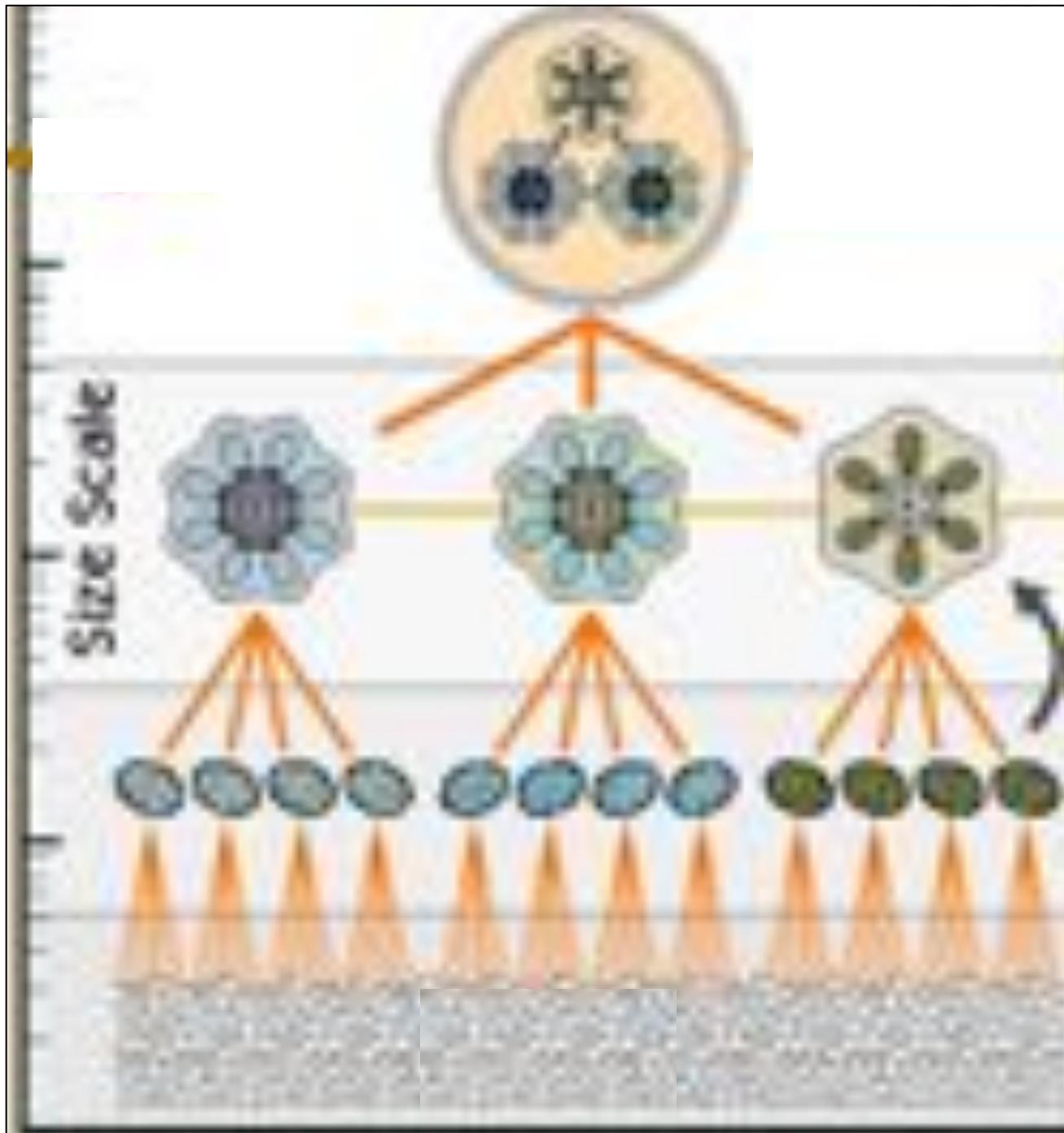
From Hinting  
Broken Components  
to Understanding  
Complex Systems

Sidney Dekker

**complex**



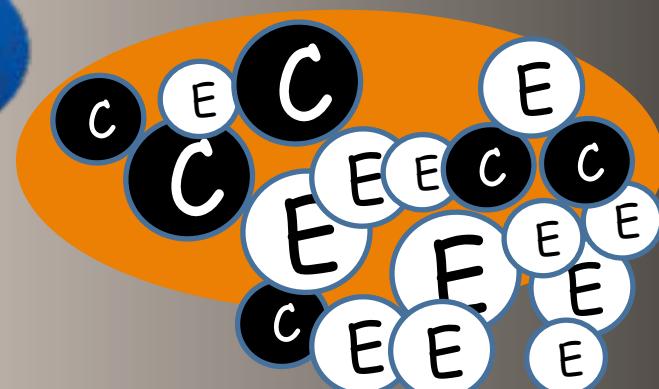
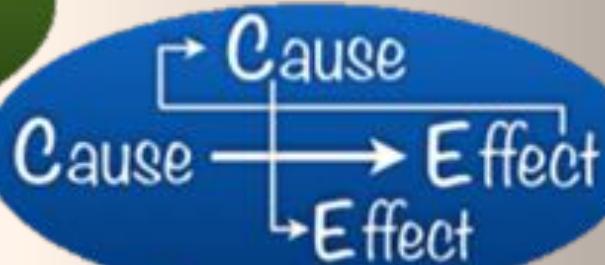
**simple**



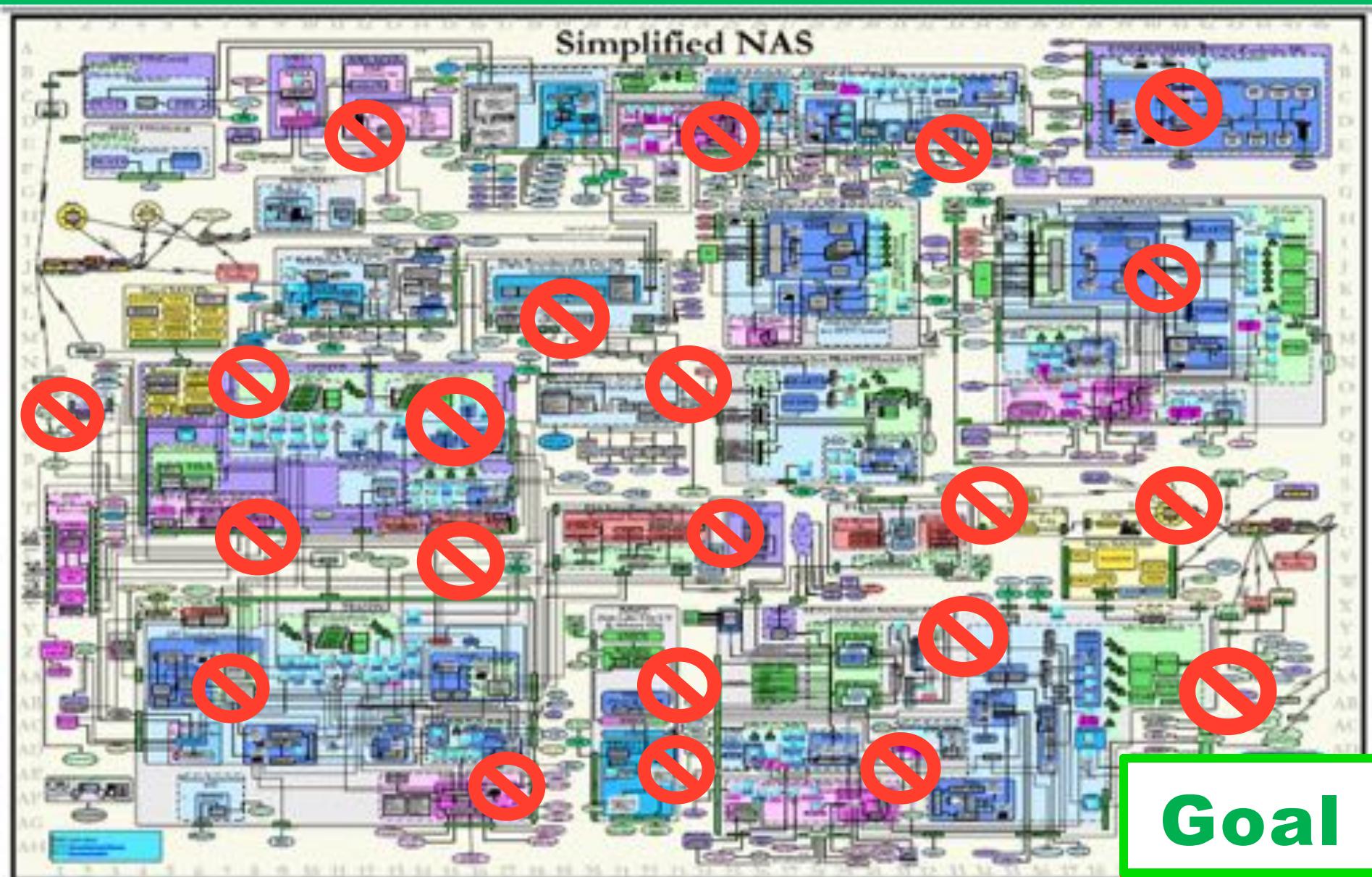
# Simple      Complicated      Complex



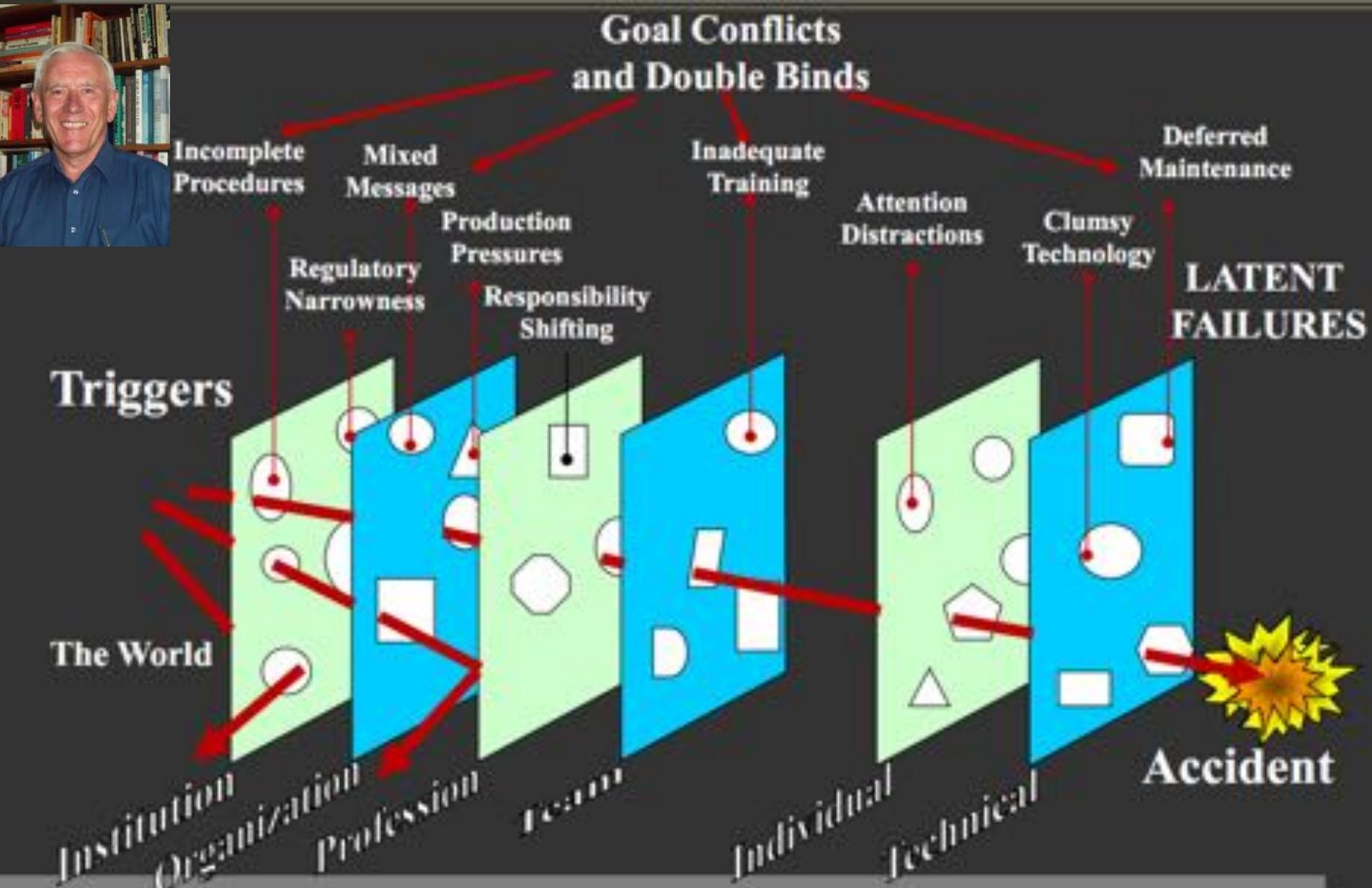
Cause > Effect



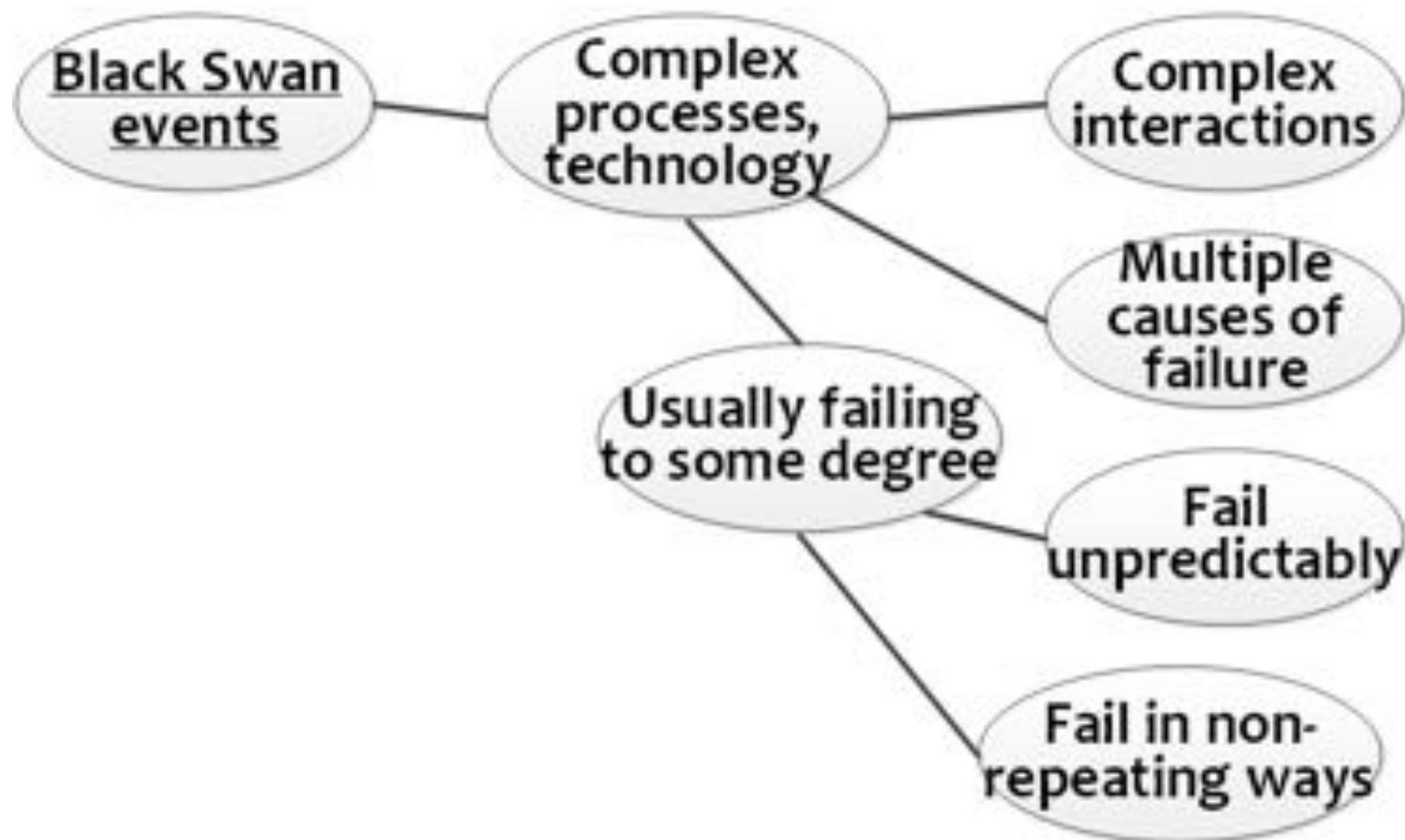
# Complex operations



# Swiss Cheese model (Orlandella/Reason)



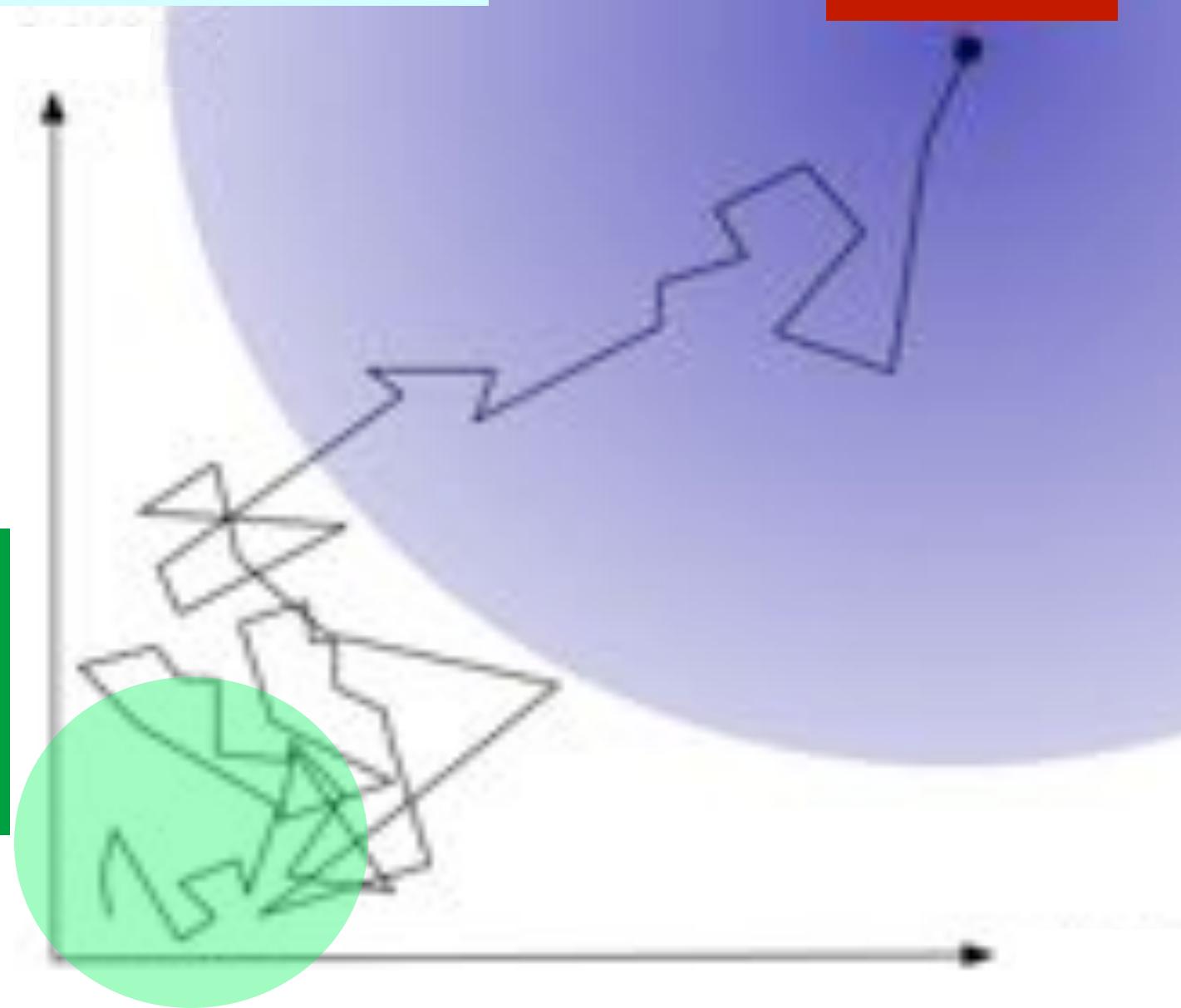
# Summary



# Drift into failure

Failure

Intended  
operating  
range

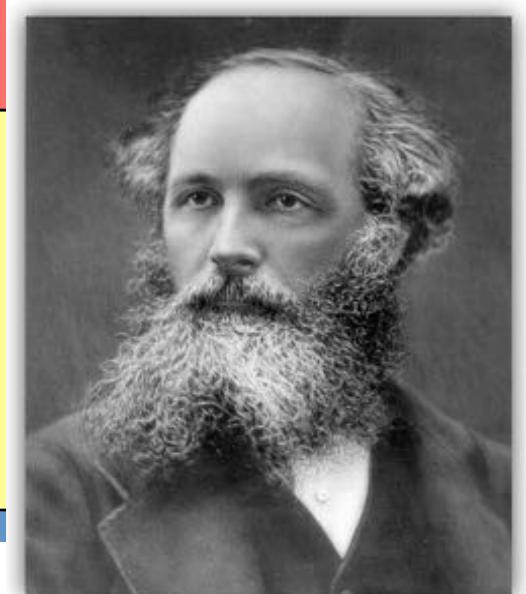
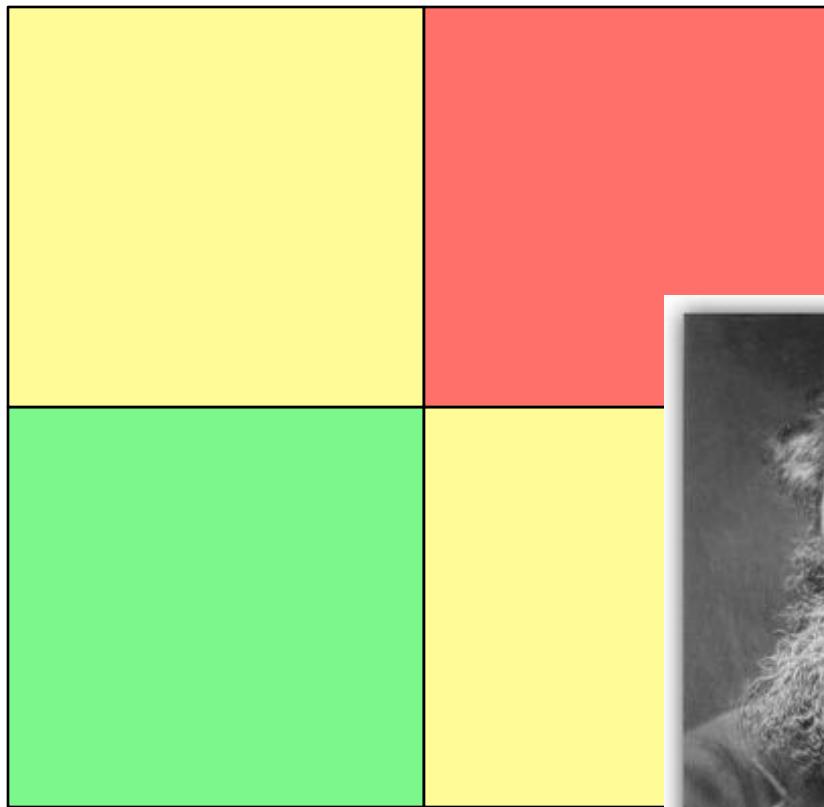


**System:**  
Linear      Complex

**Coupling:**

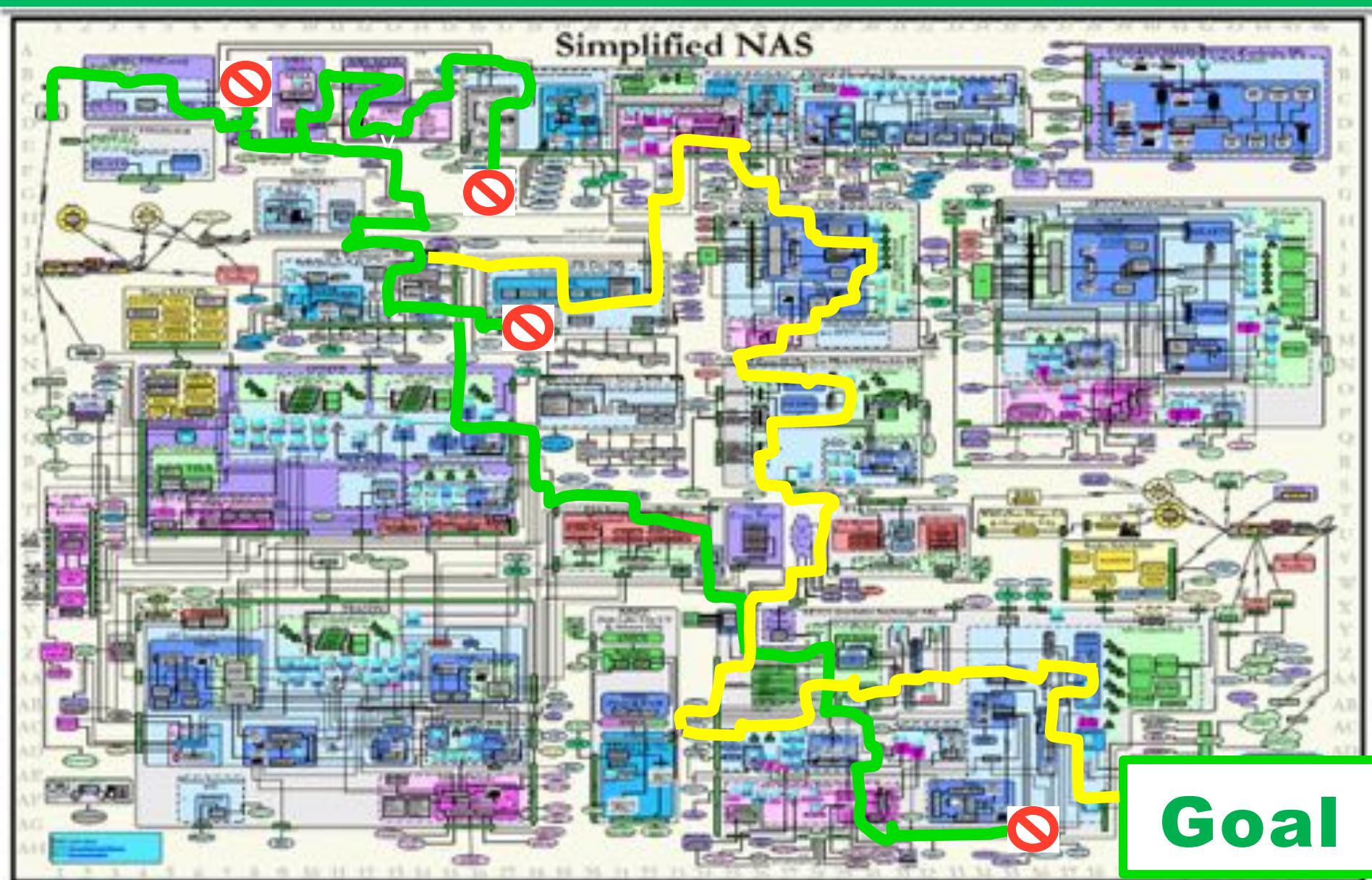
Tight

Loose



**James Clerk Maxwell**  
**(1839-1879)**

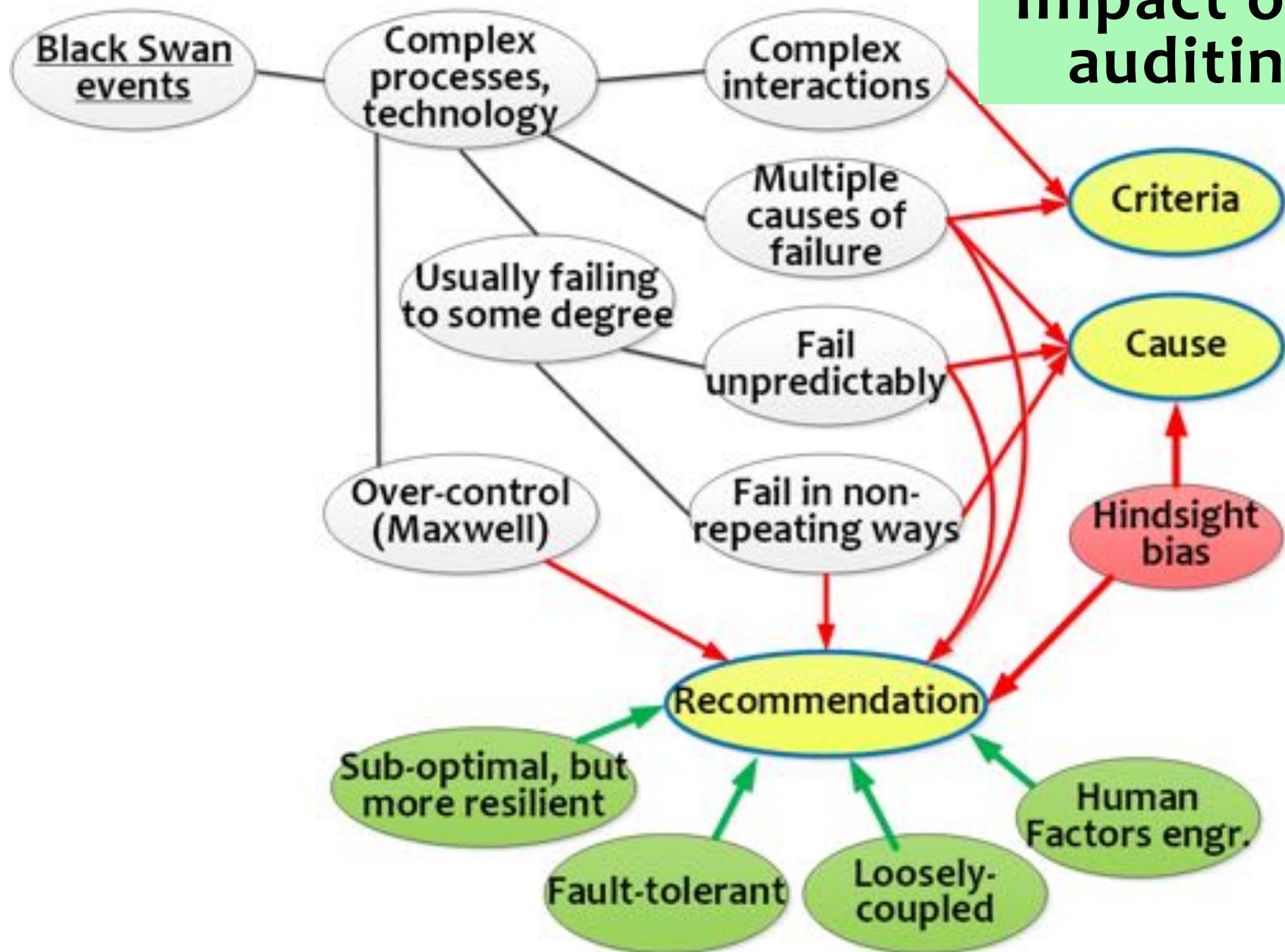
# Complex operations

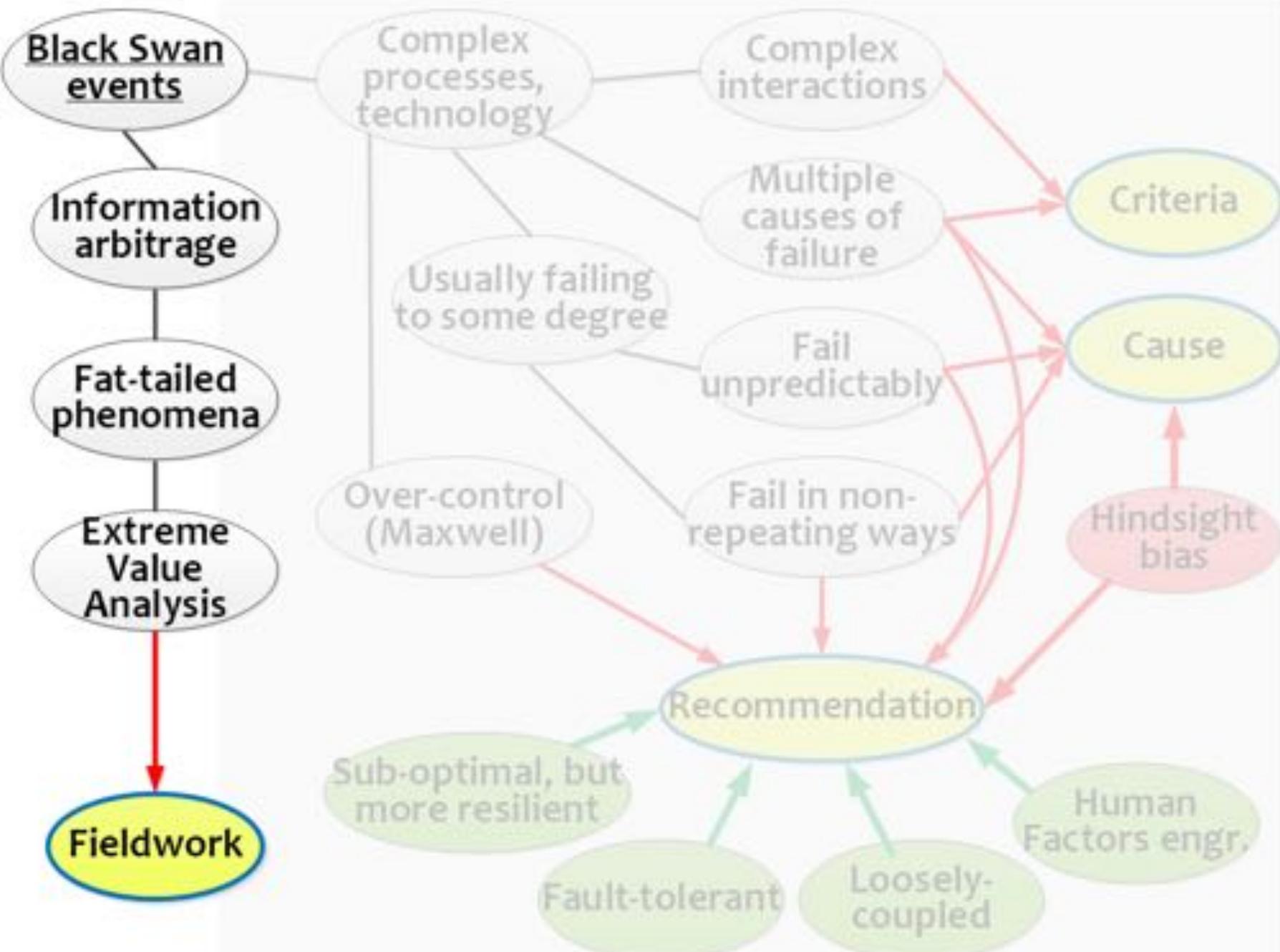


insert slide:

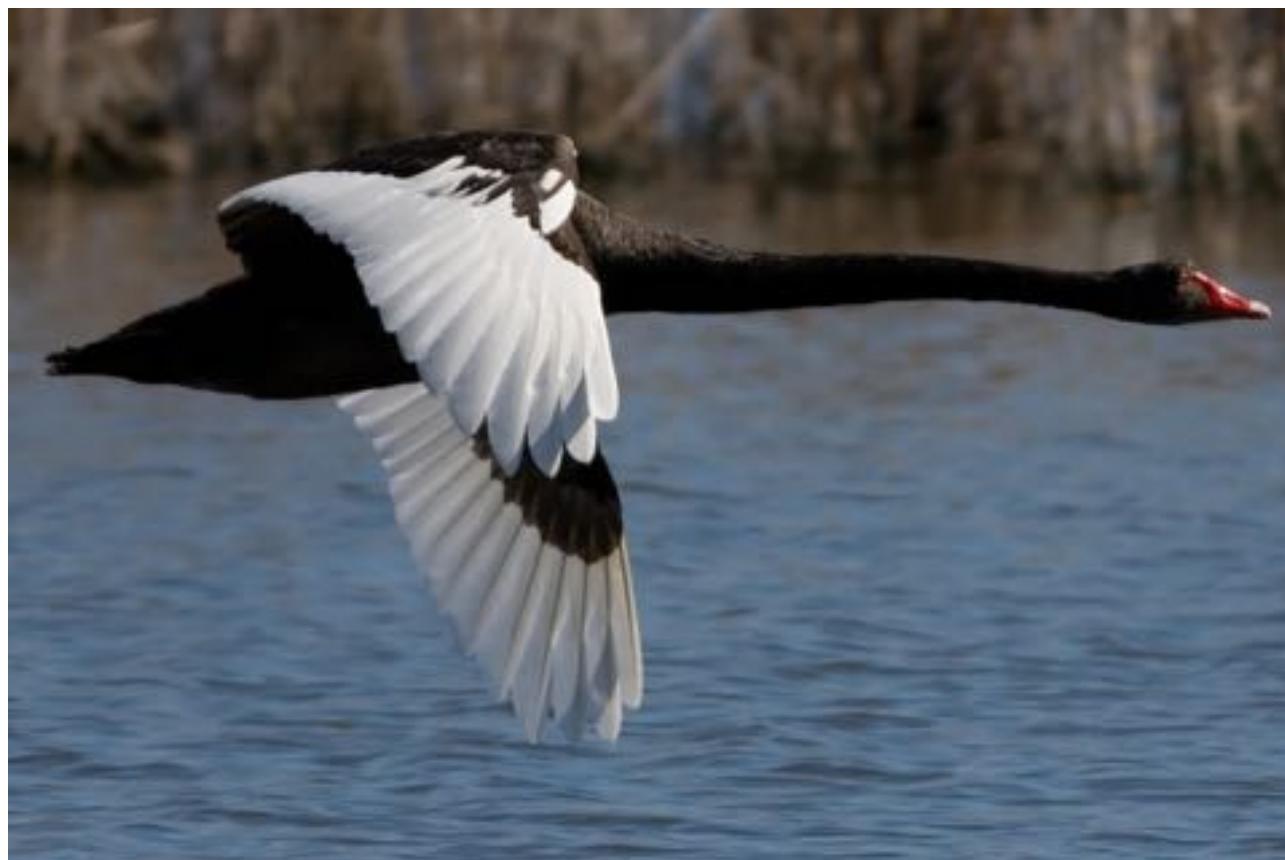
- post-accident attribution to human error
- hindsight bias
- human factors engineering
- photo of employee

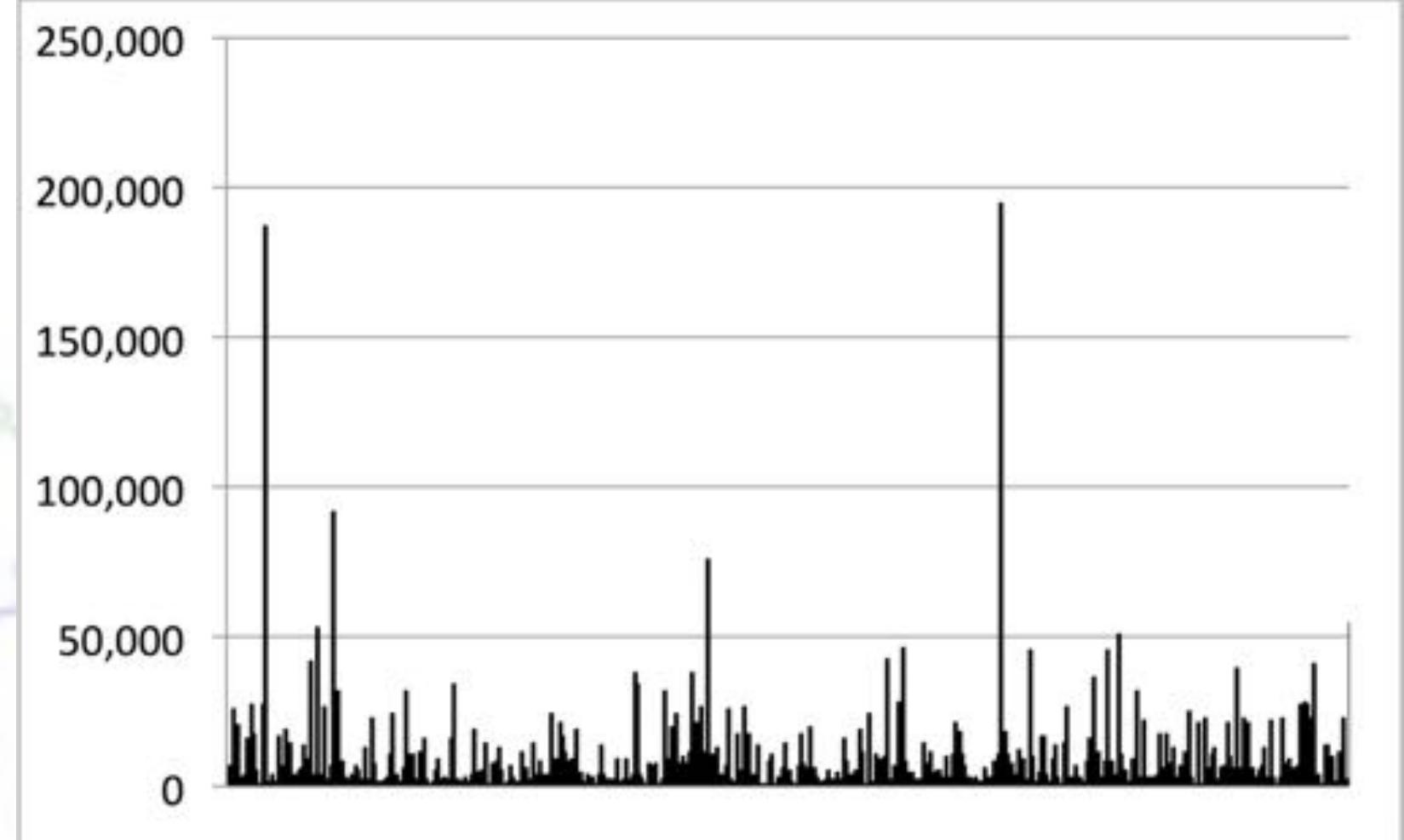
# Impact on auditing



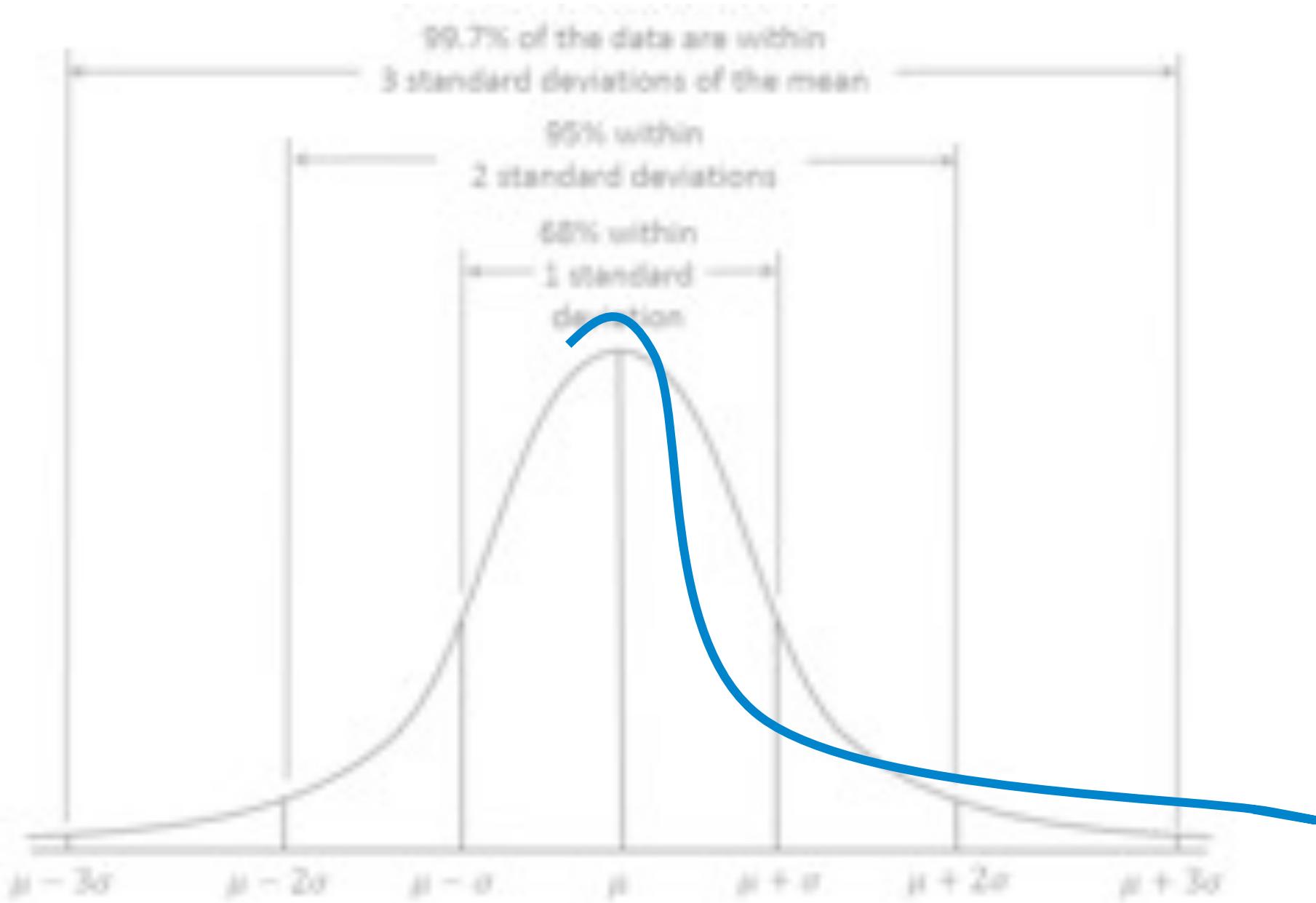


# Extreme Value Analysis

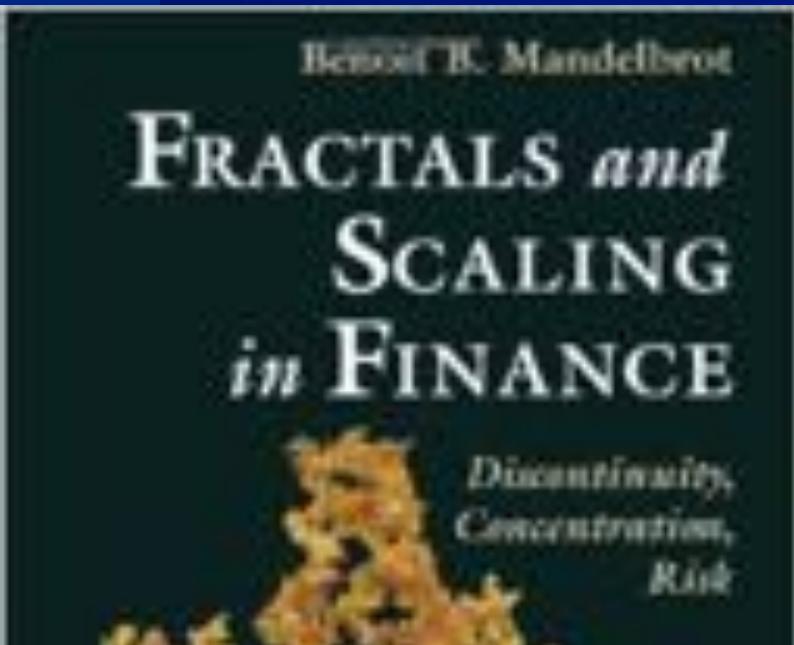




# ... often doesn't work



# Benoit Mandelbrot



- Cotton prices exhibit power-law behavior
- 80-20 rule of thumb
- Fractal phenomena





**Vilfredo Pareto, 1848-1923**

**Generalized Pareto Distribution**

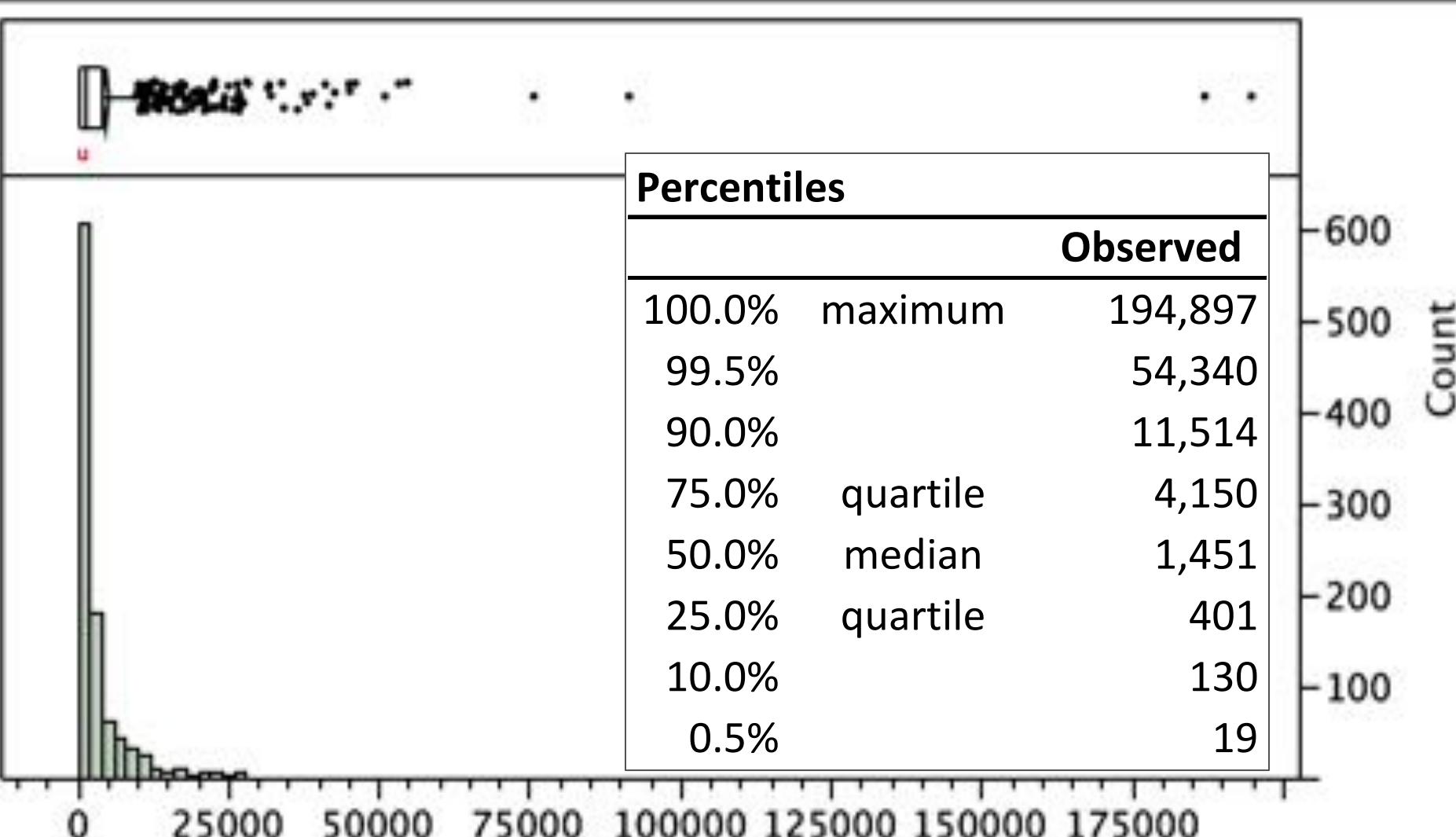
**80-20 rule**

# seven years of records:

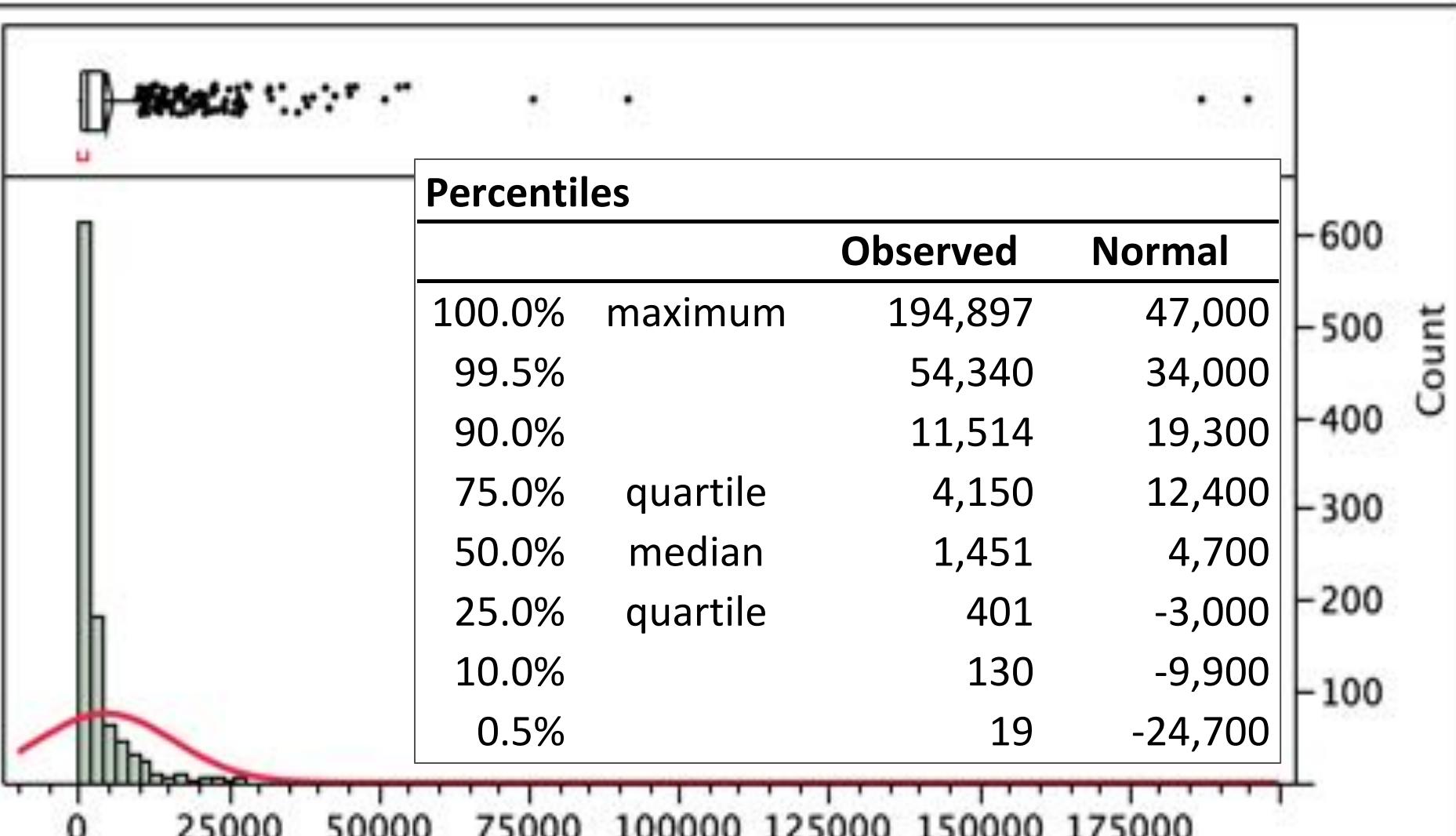
*Frequency distribution: self-insurance claims 2006-2012*

Range	2006	2007	2008	2009	2010	2011	2012	Total
10,000	63	99	126	149	183	165	159	944
20,000	5	6	7	11	14	12	16	71
30,000	1	5	4	4	9	6	5	34
40,000	1		1	1	4	2	1	10
50,000			1	2	1	1	1	6
60,000		1	1		1			3
70,000								0
80,000						1		1
90,000								0
100,000		1						1
⋮								
170,000								0
180,000								0
190,000						1	1	1
200,000				1				1

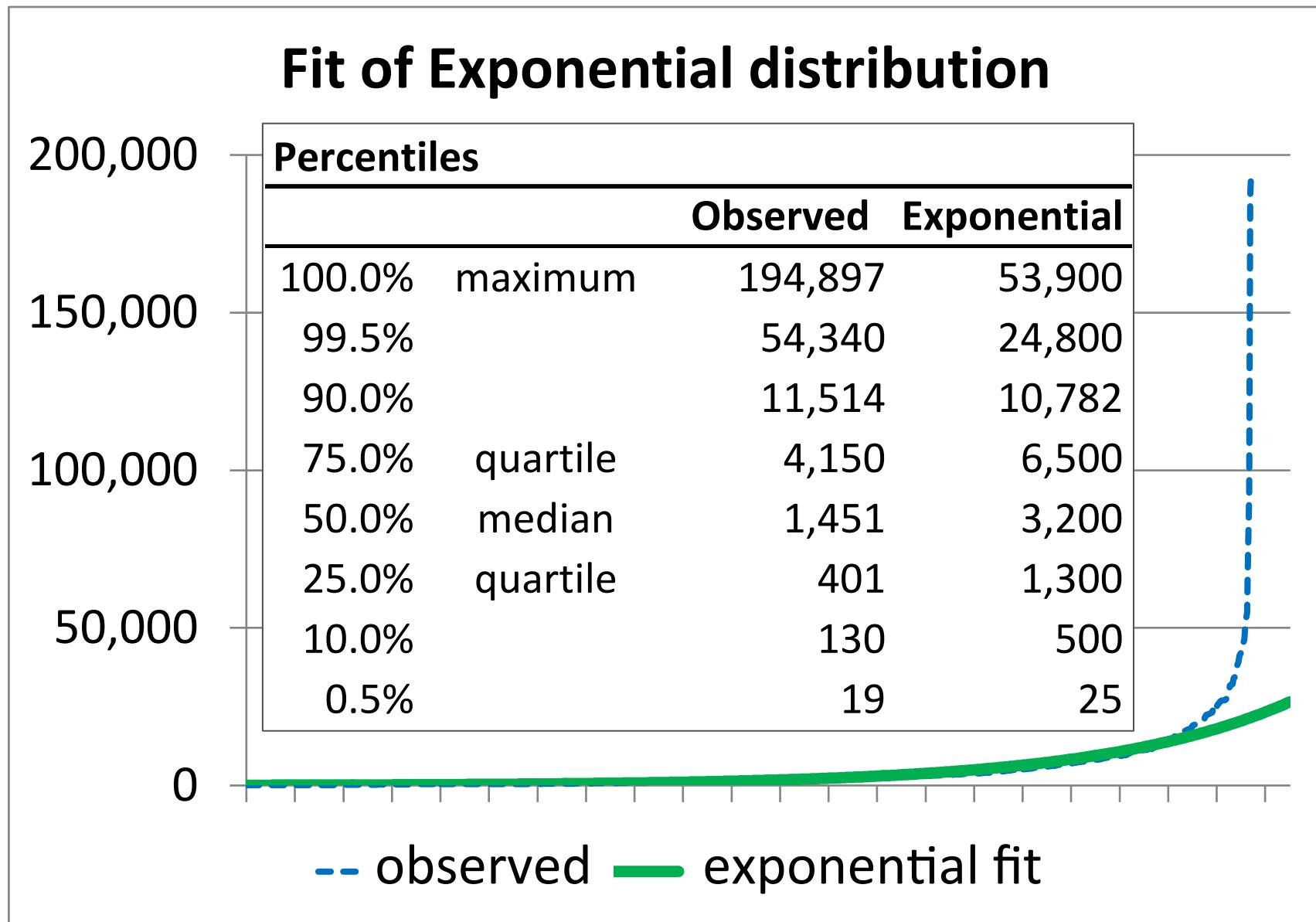
# seven years of records:



# normal distribution fits poorly...



# exponential distribution, the same...





# Generalized Pareto Distribution

Vilfredo Pareto  
1848-1923

three variables:

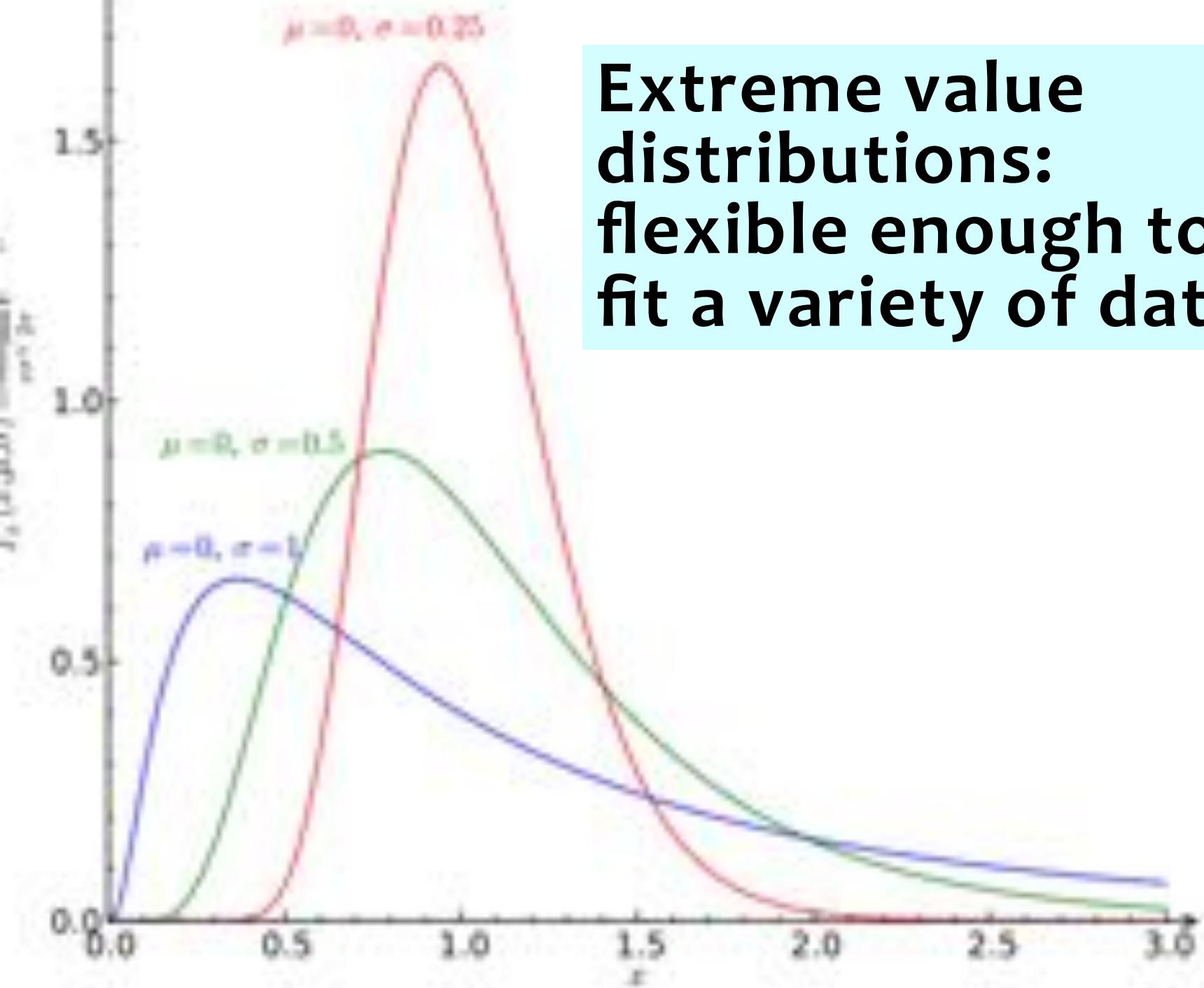
$\mu$  = location

$\sigma$  = scale

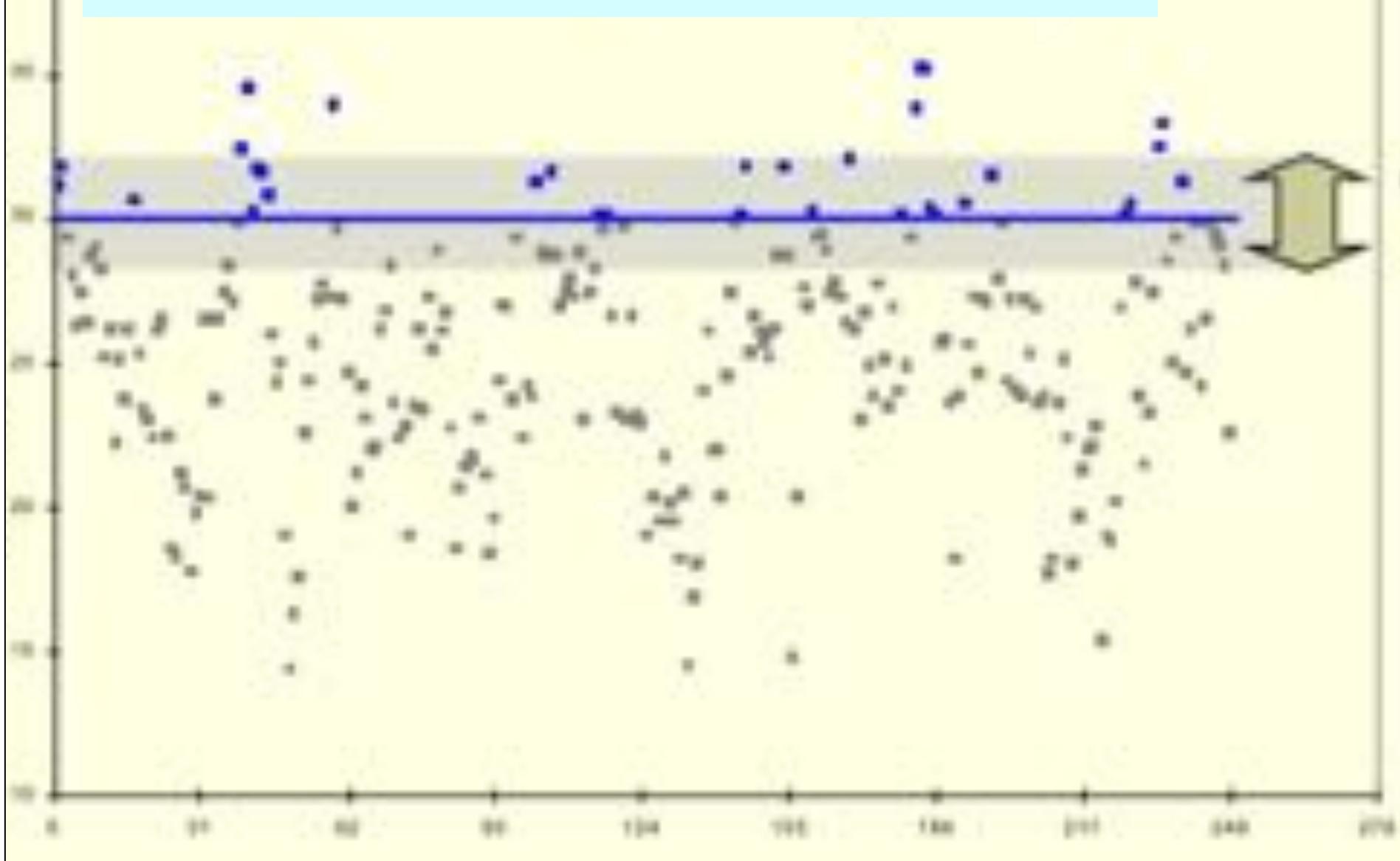
$k$  = shape

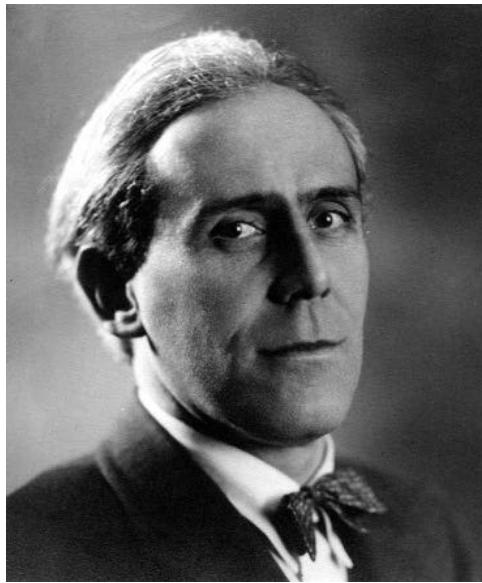
$$\frac{1}{\bar{\sigma}} \left( \frac{1 + k}{\sigma} \right)^{-1 - 1/k}$$
$$\frac{1}{\bar{\sigma}} \left( \frac{(x - \mu)}{\sigma} \right)^{-1 - 1/k}$$

**Extreme value distributions:  
flexible enough to  
fit a variety of data**



# peaks-over-threshold (PoT): all records above a threshold





Emil Gumbel



Maurice Fréchet



Weibull

(i) *Gumbel (type I)*:  $\Lambda(x) = \exp\{-\exp(-x)\}, x \in \mathbb{R};$

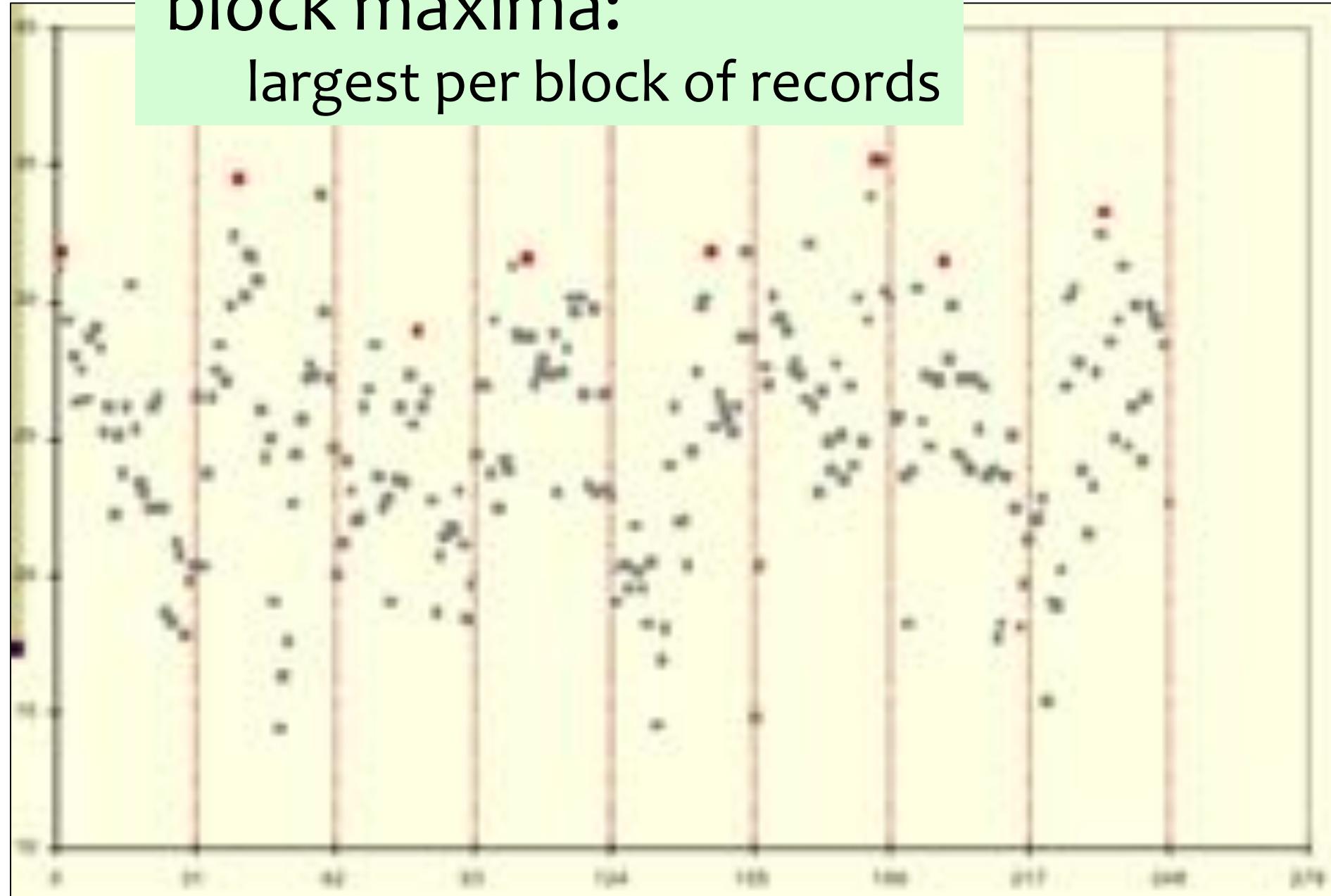
(ii) *Fréchet (type II)*:  $\Phi_\alpha(x) = \begin{cases} 0, & x \leq 0; \\ \exp\{-x^{-\alpha}\}, & x > 0, \alpha > 0; \end{cases}$

(iii) *Weibull (type III)*:  $\Psi_\alpha(x) = \begin{cases} \exp\{-(-x)^\alpha\}, & x \leq 0, \alpha > 0; \\ 1, & x > 0. \end{cases}$

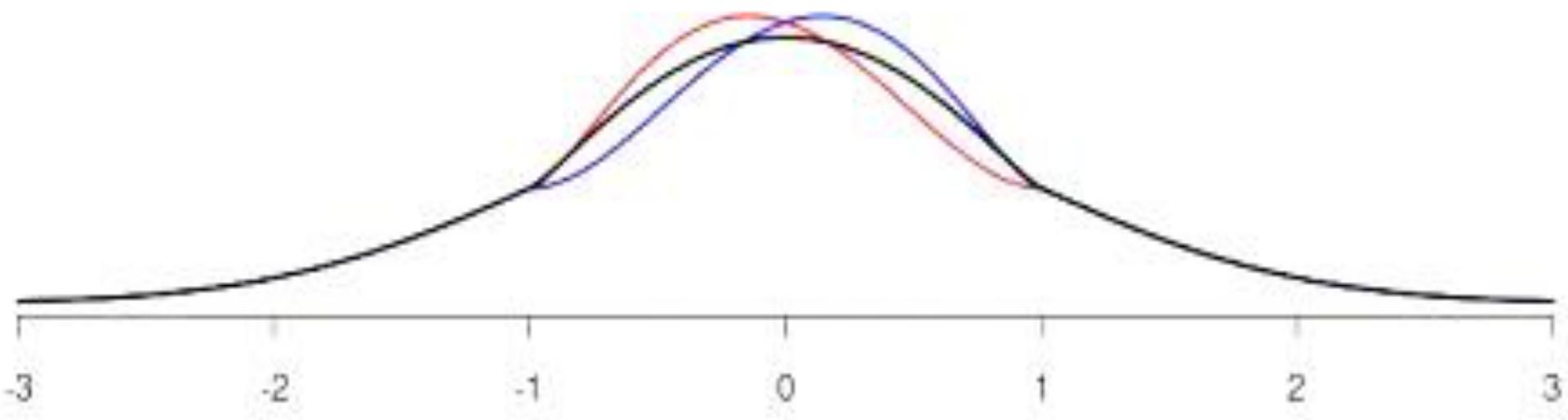


Generalized Extreme Value Distribution

block maxima:  
largest per block of records



# Key idea: the tail has its own “child” distribution



- *No assumptions are made about the non-extreme observations*



# EVA in practice, part I quantifying exposure & probability

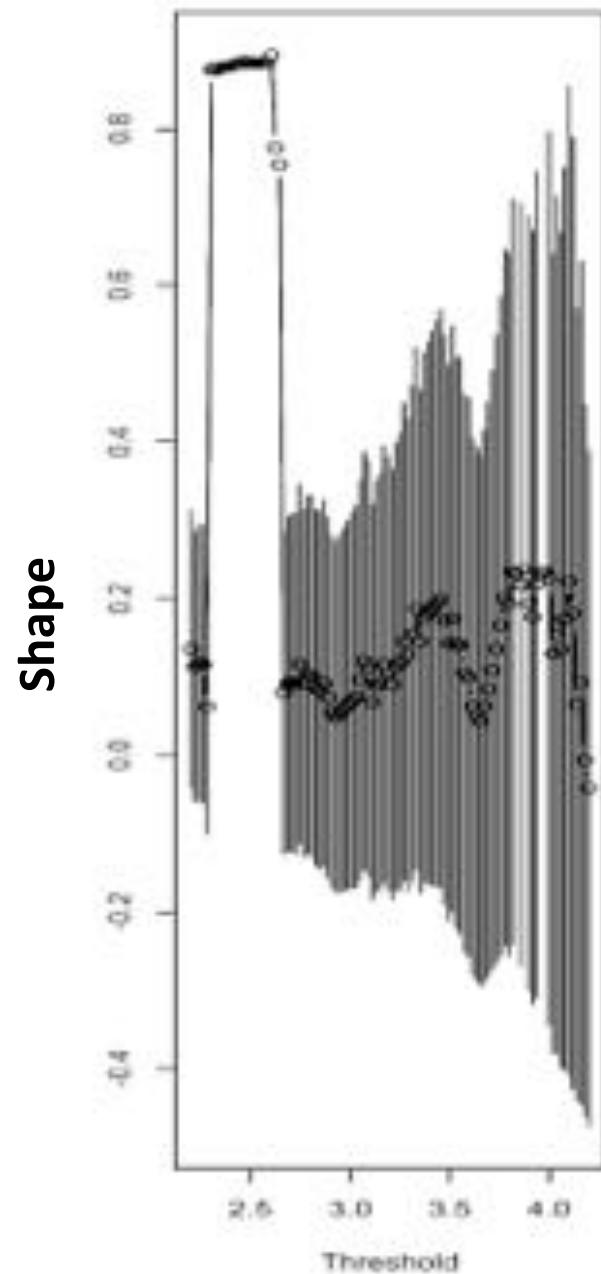
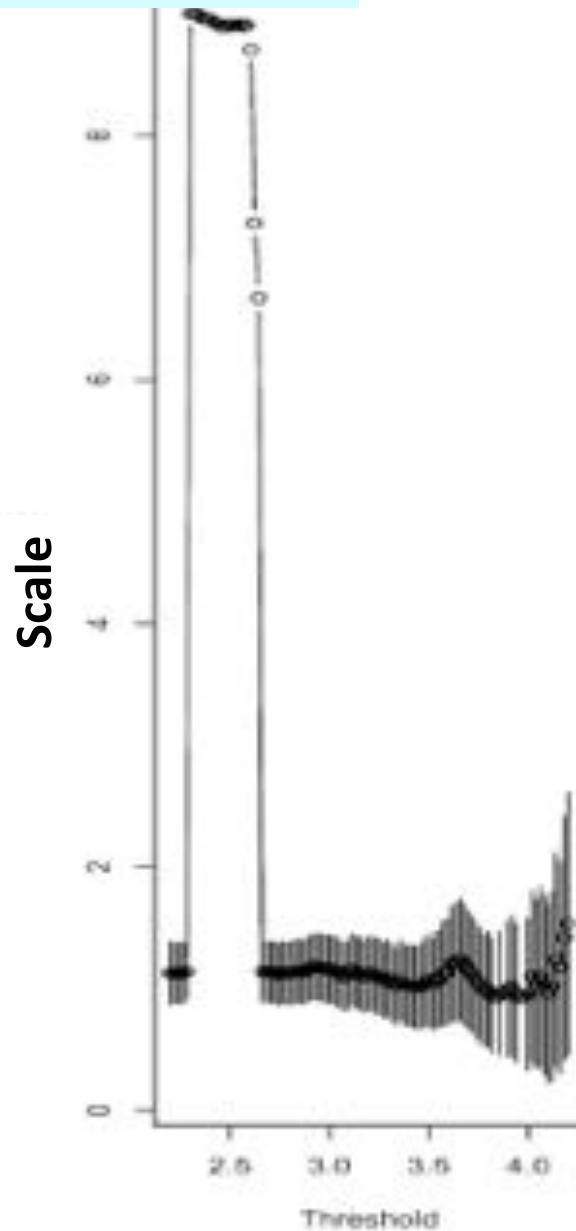
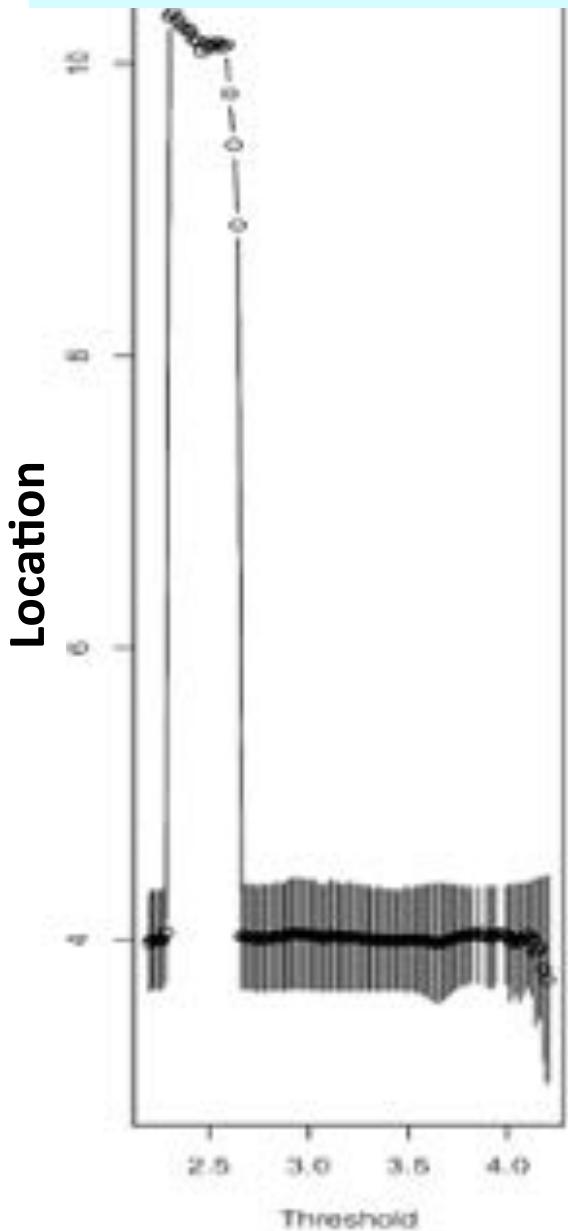




## EVA in practice, part I

- Is EVA suitable for the data?
- What threshold is best?
- What values for location, scale, shape?
- What losses for range of probabilities?

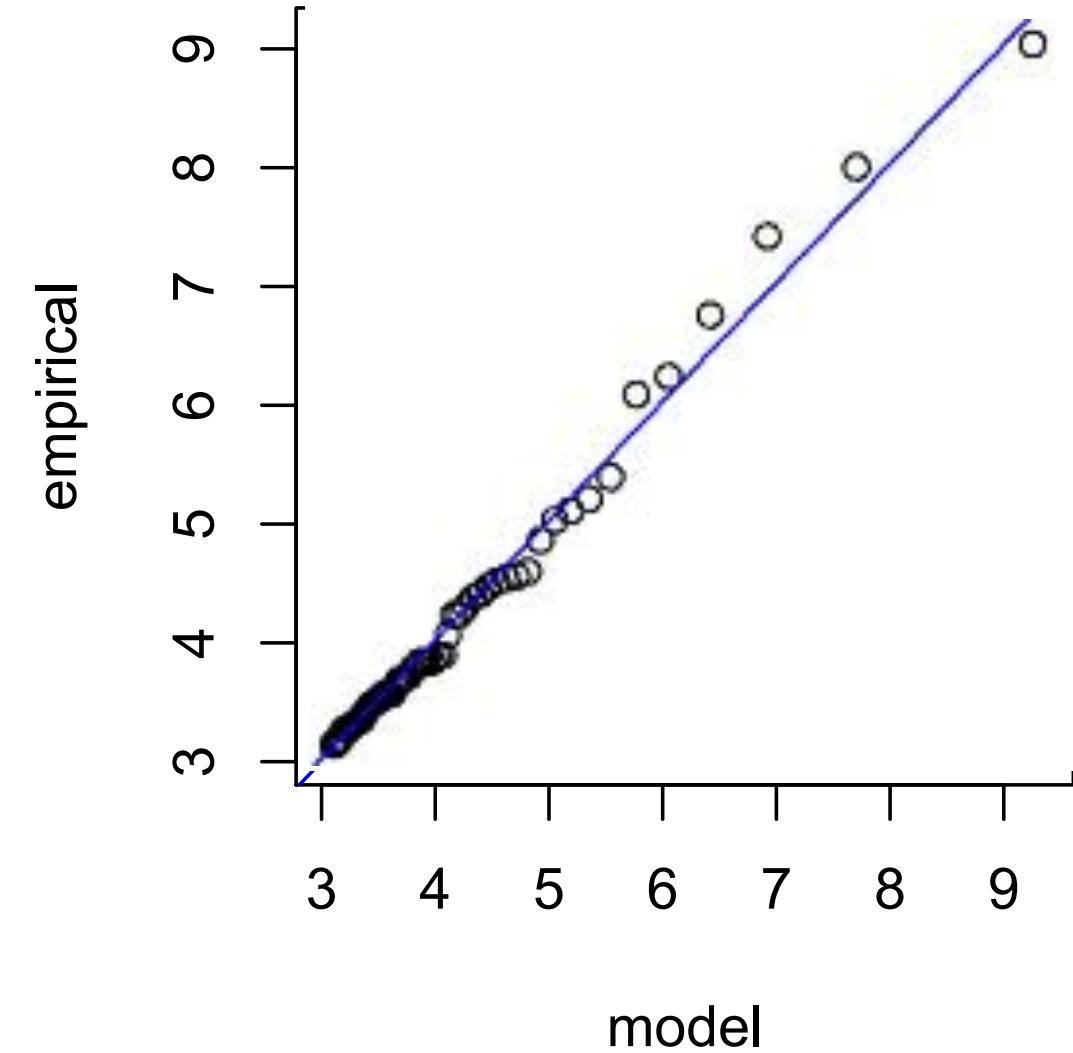
# Threshold selection:



# Diagnose fit:



## Quantile Plot



# Best-fit results:

```
> current1 <- pp.fit(current, 2375, npy = 153)
```

threshold      2,375

nexc            181

nllh            2,733.23

mle            71,115.9    44,033.9    0.574

se             3,017.5    1,775.6    0.025



© Akbar Khan



# EVA in practice, Part II

Compute exposure  
x probability:

Probability of annual occurrence	Return Period (years)	-95% c.i.	"most likely"	+95% c.i.
90%	1.1	70,000	76,000	82,000
40%	2.5	113,000	124,000	136,000
20%	5.0	168,000	188,000	209,000
13%	7.5	210,000	238,000	269,000
10%	10.0	246,000	282,000	321,000



# EVA in practice, Part II risk mitigation



# Annual reserve funding:

Annual contribution for reserve:			
Return Period (years)	-95%c.i.	"most likely"	+95%c.i.
1.1	63,000	68,400	73,800
2.5	45,200	49,600	54,400
5.0	33,600	37,600	41,800
7.5	27,993	31,725	35,858
10.0	24,600	28,200	32,100
Annual expense:			
	194,393	215,525	237,958

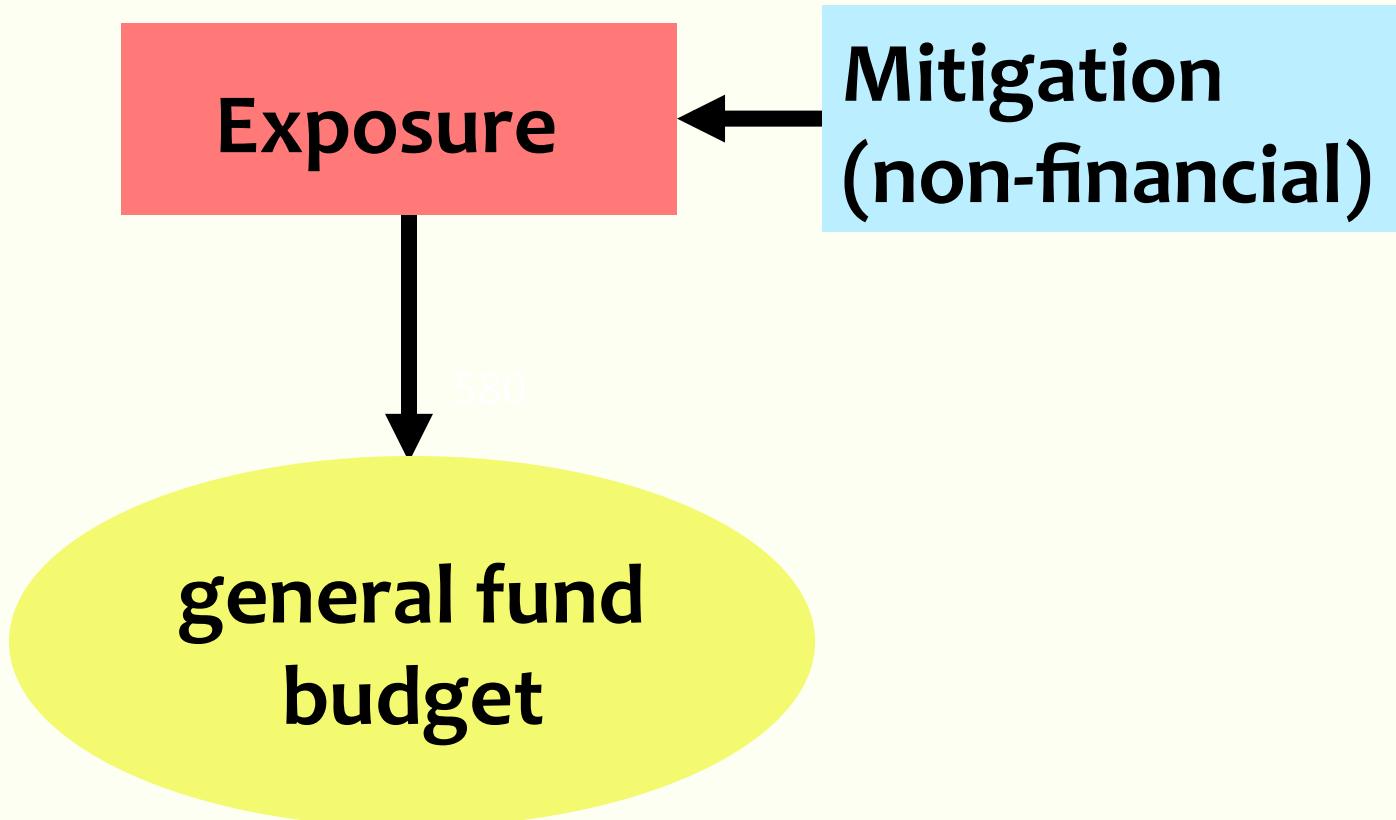
**Frequency distribution: count of offsite services 2006-2012**

Range	Count								Avg. per year
	2006	2007	2008	2009	2010	2011	2012	Total	
10,000	63	99	126	149	183	165	159	944	134.86
20,000	5	6	7	11	14	12	16	71	10.14
30,000	1	5	4	4	9	6	5	34	4.86
40,000	1		1	1	4	2	1	7	1.43
50,000			1	2	1	1	1	6	0.86
60,000		1	1		1			3	0.43
70,000					1			0	0.00
80,000						1		1	0.14
90,000								0	0.00
100,000							1	1	0.14
⋮									
170,000								0	0.00
180,000								0	0.00
190,000							1	1	0.14
200,000					1			1	0.14

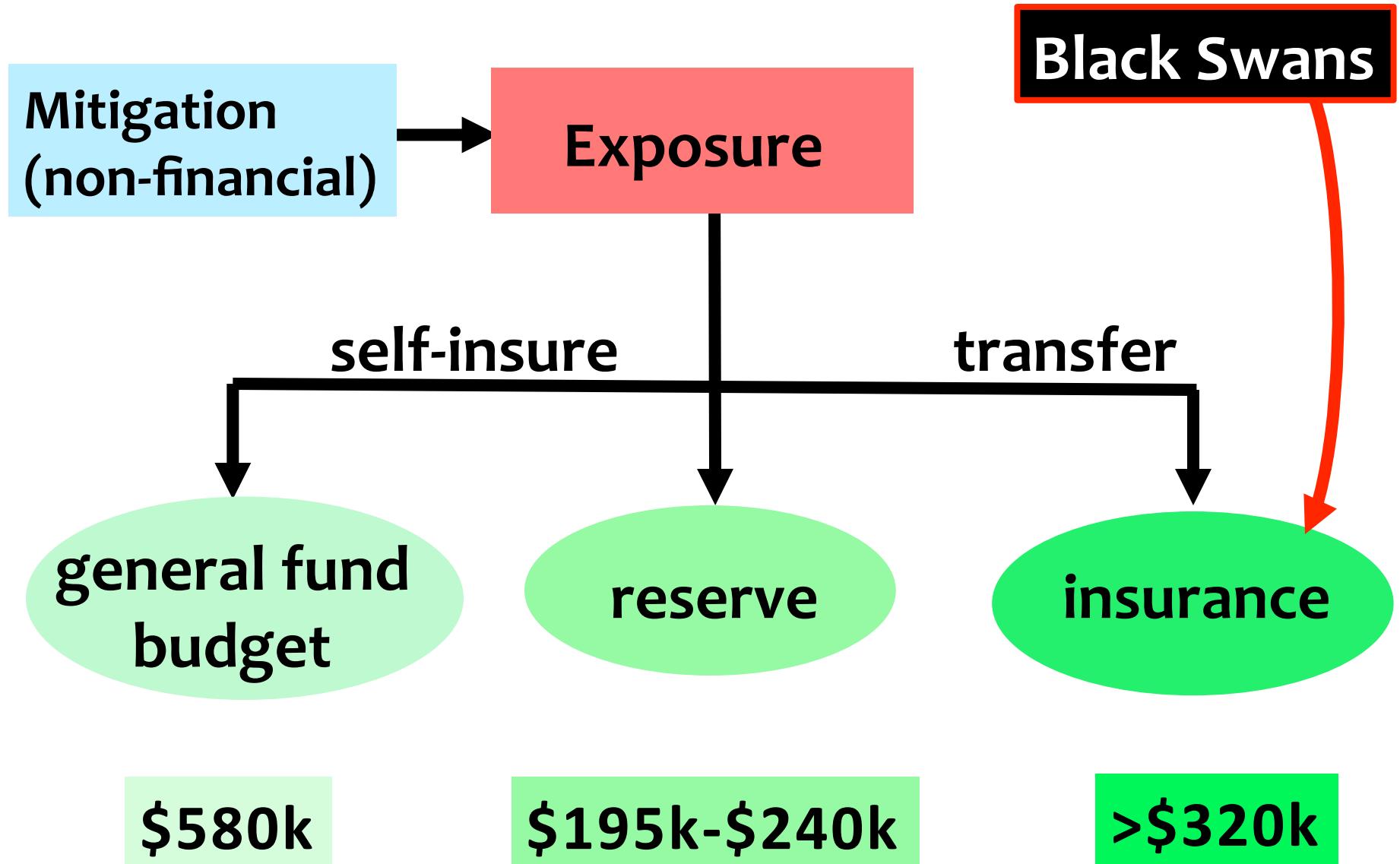
$p \geq 100\%$  funded from general fund

# Pre-audit risk management:

**100% self-insured**

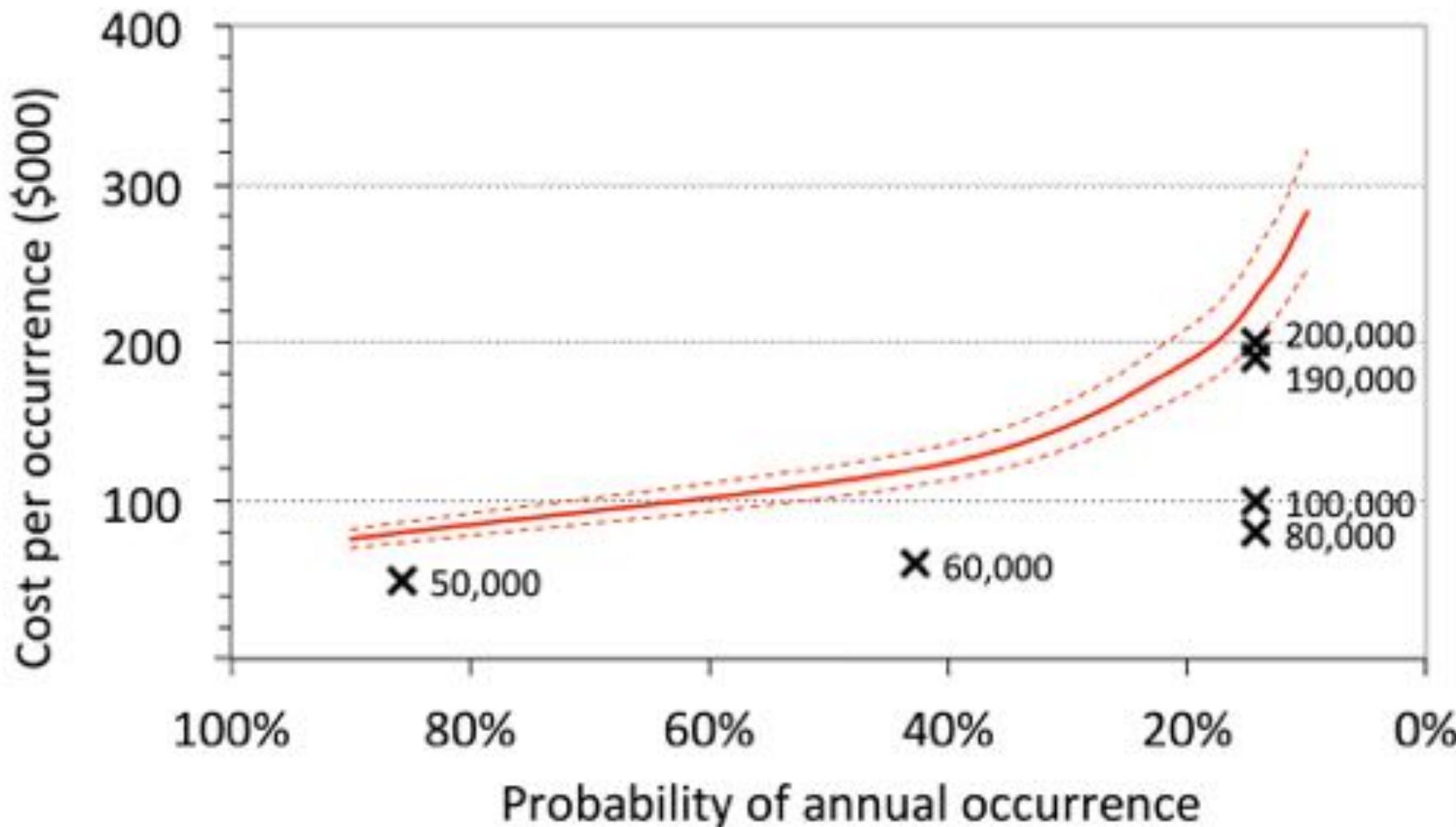


# Tiered risk management:



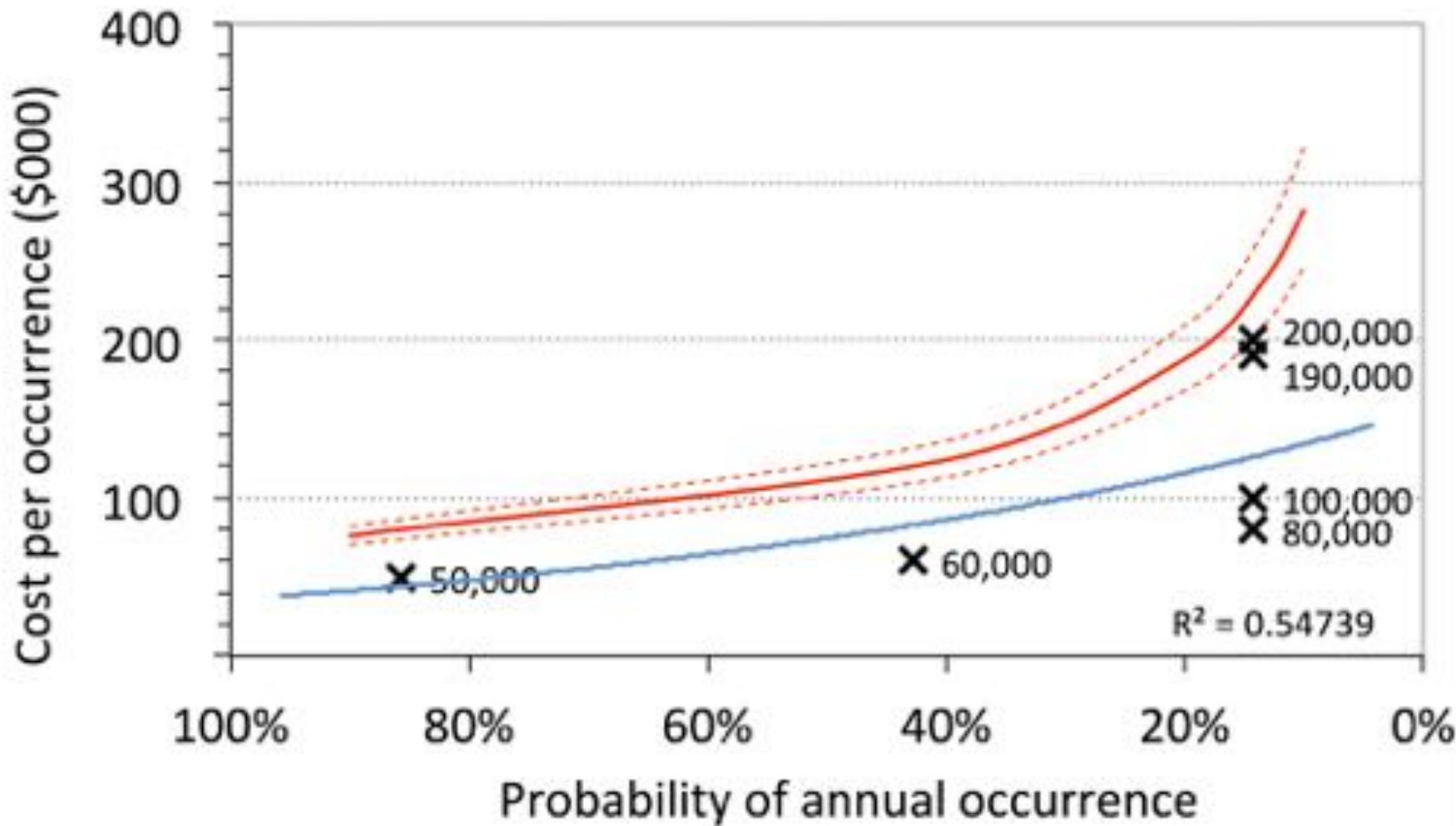
# How well did it work?

High-cost healthcare events: EVA & observed



# ... compared to what?

High-cost healthcare events: EVA & observed



# only six “r” commands required

**Import data file:**

```
working-filename <- scan(file = "filename.ext", sep = ",")
```

**Test whether EVA is suited for the data:**

```
lmomplot(working-filename, identify = TRUE)
```

**Test fit over a range of thresholds:**

```
pp.fitrange(working-filename, 2, 5, npy = 192.6, nint = 100)
```

**Estimate parameters for the selected threshold:**

```
results-file <- pp.fit(working-filename, 2.7878, npy = 192.6)
```

**Diagnostic plot of fit:**

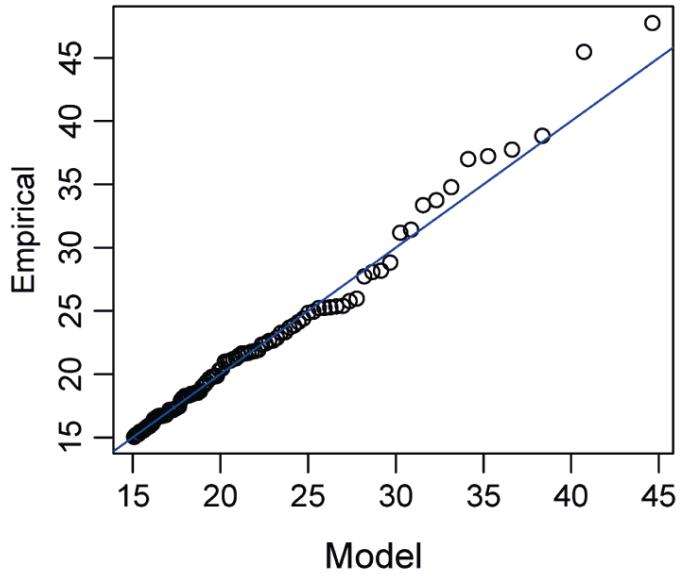
```
pp.diag(results-file)
```

**Determine exposure for a given probability:**

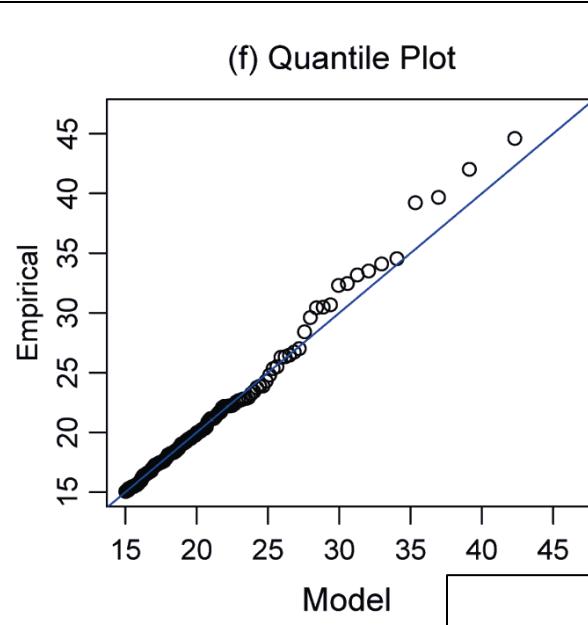
```
qgpd( $q, \mu, \sigma, \xi$ )
```

# Diagnose fit, other fit examples:

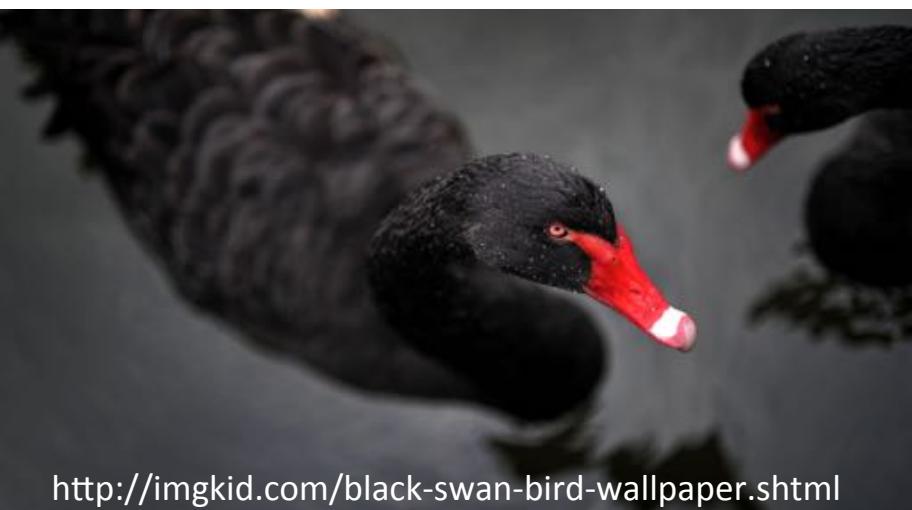
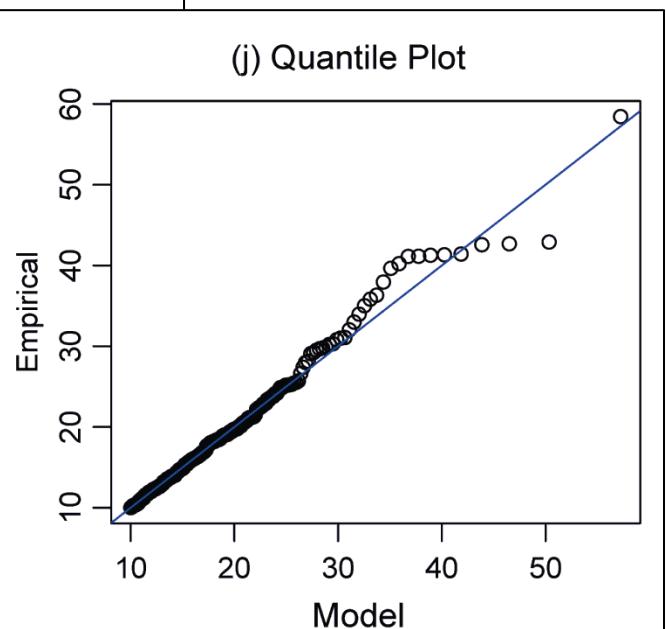
(b) Quantile Plot

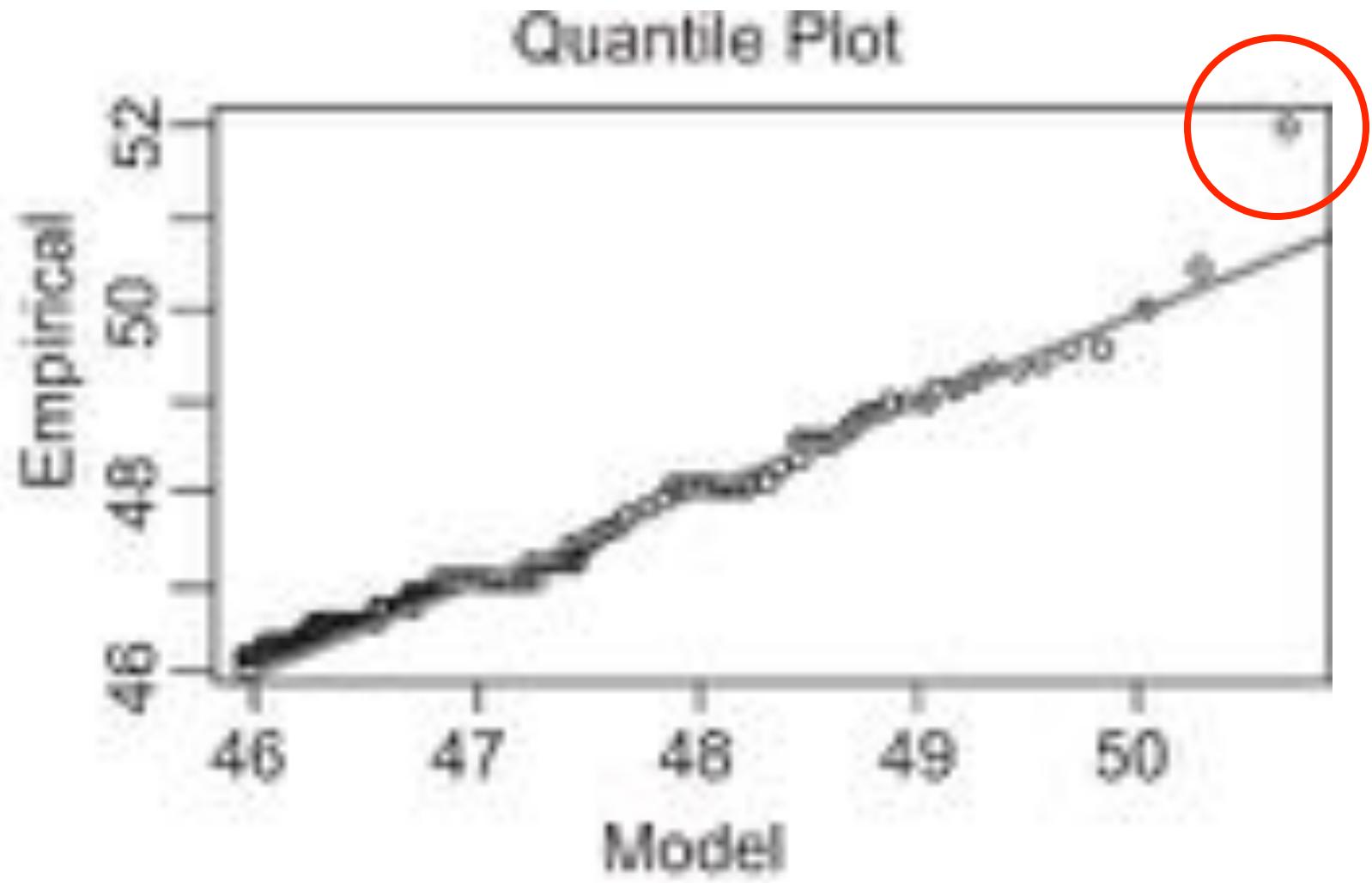


(f) Quantile Plot

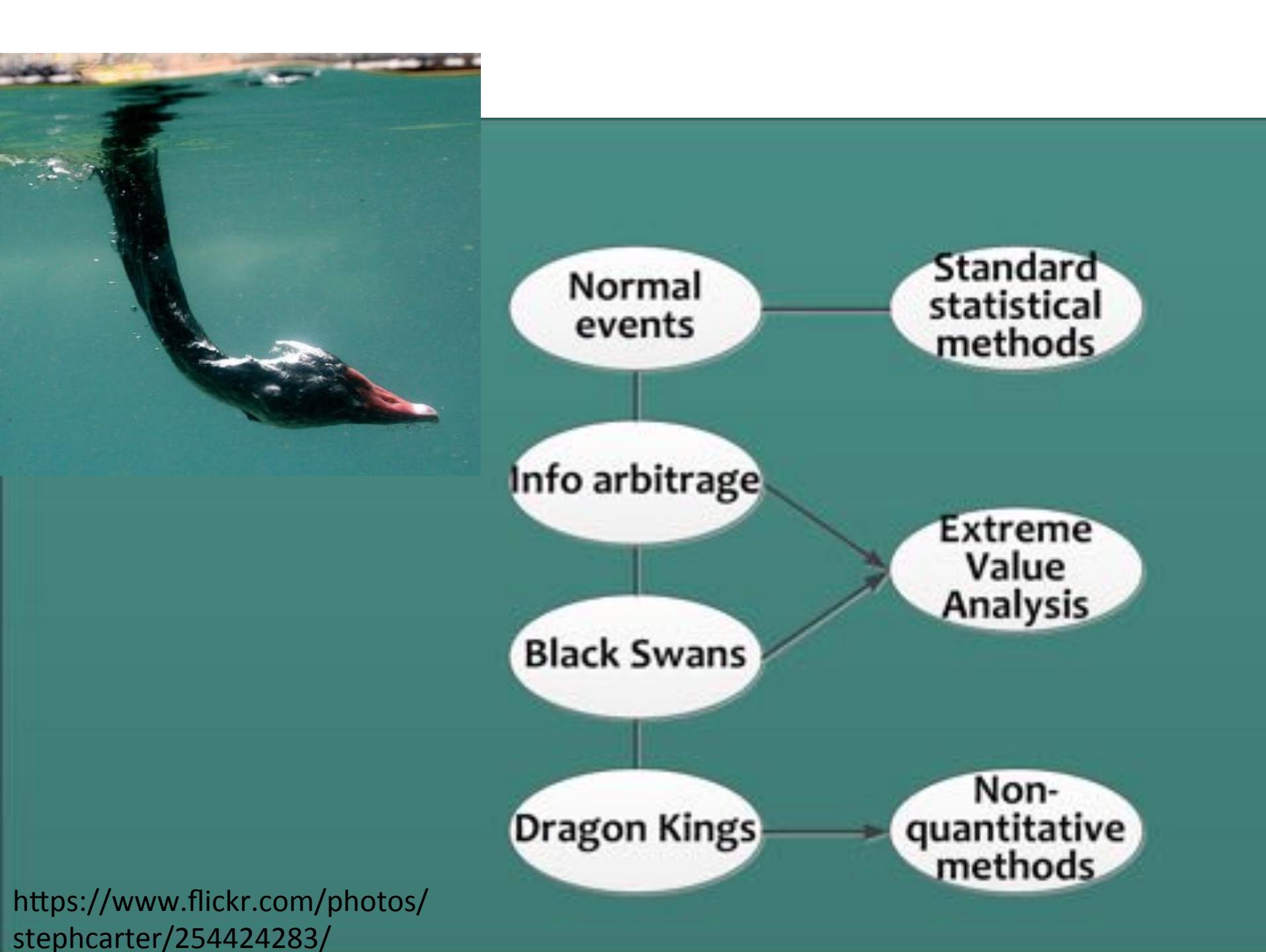


(j) Quantile Plot

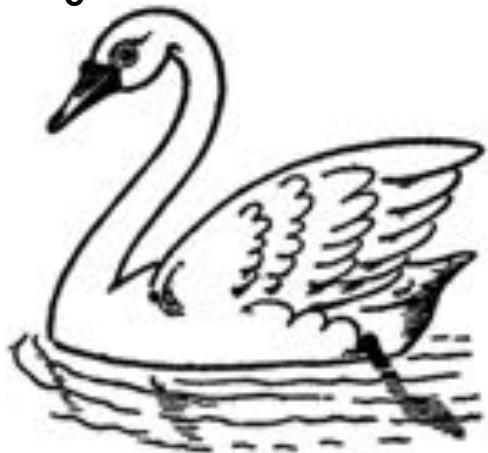




Cueto ORG, Soto NS, Quintero Núñez MQ, Benítez SO, Limón NV (2013) Extreme temperature scenarios in Mexicali, Mexico under climate change conditions. *Atmósfera* 26(4):509-520 doi: 10.1016/S0187-6236(13)71092-0



# Outline:



**Wrap and Q & A**

**Nassim Taleb's The Black Swan**

**Accidensts, safety, complexity**

**Extreme Value Analysis, theory**

**EVA in practice, program auditing I**

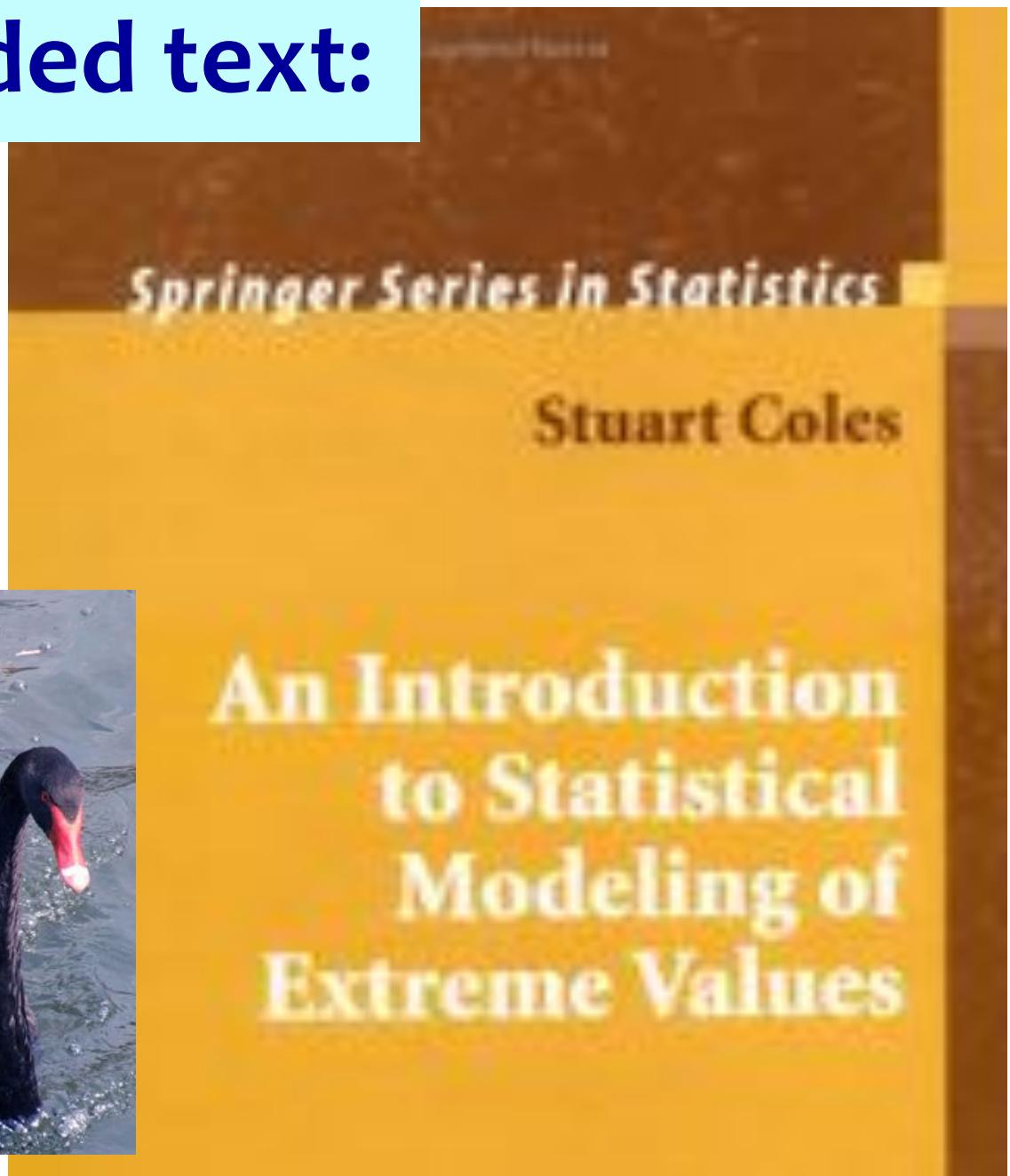
**EVA in practice, program auditing II**

# Tips:

- *The numbers only tell part of the story*
- *Focus on the confidence interval*
- *Be mindful of the black swans*
- *Don't over-extend*
- *Be conservative*
- *Trust the method*



# Recommended text:



credit: Sergis V. Duarte, via [wikimedia.org](https://commons.wikimedia.org) 12/2006

## References

- Bensalah Y (2000) *Steps in Applying Extreme Value Theory to Finance: A Review*. Research and Risk Management Section, Financial Markets Department, Bank of Canada. Ottawa, Ontario, Canada p 9
- Brüggen J (2012) *The global voice of ATM ICAO European Regional Runway Safety Seminar* Amsterdam, The Netherlands 8 March 2012 <http://slideplayer.com/slide/1448937/>
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- Mohtadi H, Murshid A (2005) *Assessing the Risk of Terrorism Using Extreme Value Statistics*. Proceedings of the Institute of Food Technologists' First Annual Food Protection and Defense Conference, October 27, 2005

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Perrow C (1999) *Normal Accidents: Living with High Risk Technologies*. Princeton University Press 464 pps

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Taleb N (2010) *The Fourth Quadrant: A Map of the Limits of Statistics*. edge.org  
<http://www.slideshare.net/MediaCorpInternational/130221-0850-david-bond-thiess>  
[www.nytimes.com/2007/04/22/books/chapters/0422-1st-tale.html](http://www.nytimes.com/2007/04/22/books/chapters/0422-1st-tale.html)  
[www.thefullwiki.org/Black\\_swan\\_theory](http://www.thefullwiki.org/Black_swan_theory)  
[www.insead.edu/facultyresearch/faculty/documents/4\\_Quadrants-w.pdf](http://www.insead.edu/facultyresearch/faculty/documents/4_Quadrants-w.pdf)  
[www.nakedcapitalism.com](http://www.nakedcapitalism.com)  
[www.coverclock.blogspot.com](http://www.coverclock.blogspot.com)  
[www.zoonek2.free.fr/UNIX/48\\_R/07.html](http://www.zoonek2.free.fr/UNIX/48_R/07.html)  
[www2.etf.com/publications/journalofindexes/joi-articles/24230-what-is-risk.html?  
showall=&fullart=1](http://www2.etf.com/publications/journalofindexes/joi-articles/24230-what-is-risk.html?showall=&fullart=1)

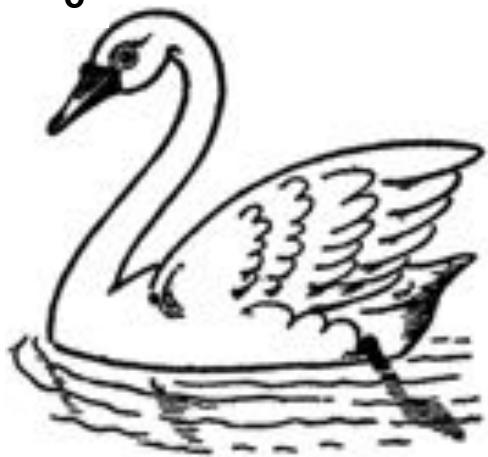
## Graphics:

Unless noted otherwise, all graphics from via [wikimedia.org](https://commons.wikimedia.org)

Title slide: <https://www.flickr.com/photos/brucelee/4667973041/>

Camouflage: <http://www.demilked.com/animal-camouflage/>

6



# Outline:

Q & A